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THE TRIPLE SYSTEM OF κ PEGASI

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ABSTRACT

The triple system of κ Pegasi has long been considered outstanding because of the apparently high masses of its components, which are entirely at variance with the mass-luminosity curve. Furthermore, in the discussion of the spectroscopic system large perturbations, such as a rapid rotation of the line of apsides, changes in a and e have been claimed which, if true, would be remarkable.

A new discussion of all available visual observations leads to a new orbit for AB which, combined with the radial velocity observations, indicates normal values for the masses. Rediscussing the same radial velocity observations as in the earlier investigation, it is found that they are well represented by a single orbit of small eccentricity. The perturbations claimed previously are shown to have been a direct consequence of the use of too many decimals. Finally, some suggestions are made as to observations which could clear up the few remaining uncertainties.

For many years κ Pegasi, with its three components of anomalous mass, has presented one of the most serious discordances with the mass-luminosity relation, seemingly based upon incontestable evidence—a combination of visual and spectroscopic observations.[1] Upon closer examination, however, it is found that although the visual binary has made nearly five revolutions since its discovery by Burnham in 1880, the last visual orbit, derived by Lewis in 1905,[2] is based upon little more than one revolution. The eccentricity found by Lewis is 0.49; Burnham gave 0.40, and from the fact that Lewis remarked that the two sets of elements are practically the same, it is evident that he himself considered the orbit only preliminary. Yet it never appears to have been tested at the crucial epoch of apparent

[1] *L.O.B.*, **9**, 120, 1918. [2] *Mem. R.A.S.*, **56**, 653, 1905.

periastron, and in 1917, more than one full revolution later, Henroteau used these same elements in his discussion without further investigation. He also discussed the spectroscopic orbit of the brighter visual component, which is a short-period binary, and came to the surprising conclusion that the line of apsides of the close system revolves in the period of the visual system, 11.3 years. This was reaffirmed by him in 1928, in his article in *Handbuch der Astrophysik*, **6,** 458, where also the statement is made that variations in a or i are indicated, while furthermore the system was quoted by Beer[3] as having a rotating line of nodes. The system thus presents some similarity to the triple 13 Ceti, in which the brighter visual component is likewise a short-period spectroscopic binary, and in which Paraskevopoulos has likewise claimed a rapid rotation of the line of apsides and changes in a, i, and P. In both cases these observed perturbations were compared to similar ones caused by the sun in the moon's orbit—Henroteau even speaks of "very great similarity"—but neither investigator appears to have realized the vast gulf, mathematically speaking, that lies between the system sun-earth-moon, with its mass ratios as high as 27,000,000 to 1, and the triple stellar systems with comparable masses. Recently Slavenas[4] has investigated the stellar case of the problem of three bodies and among others has derived the following formula for the motion of the line of apsides:

$$\frac{P}{U}=\tfrac{3}{4}k\left(\frac{P}{P'}\right)^2+\tfrac{225}{32}k^2\left(\frac{P}{P'}\right)^3+\cdots,$$

where P is the period of the close, spectroscopic system; P' the period of the wide, visual system; U the period of the apsidal revolution; and k the ratio of the mass of the fainter visual component to the total mass of the system.[5] In the present case only the first term is necessary; it gives $U=3\times10^4$ years, approximately. Henroteau derives $d\omega/dt=32^\circ\pm1^\circ$ from three sets of observations only 17 years apart. In order to reach agreement, 360° was arbitrarily added to one determination and the probable error was found by the novel

[3] *Veröff. Berlin-Babelsberg*, **5,** 78, 1927. [4] *Trans. Yale Obs.*, **6,** 41, 1925.

[5] Pogo has used this formula in his discussion of 13 Ceti, in *Ap. J.*, **68,** 133, 1928, but misquotes k.

method of taking the calculated probable error of the best determination—which was undoubtedly too small—and dividing it by the interval, thus tacitly assuming the other, still more uncertain value of ω to be infinitely accurate.

It is evident that, in a manner quite similar to what I have shown to be the case with Paraskevopoulos' conclusions about 13 Ceti, the results for κ Pegasi are caused solely by the use of too many decimals. Considering the uncertain state of the visual orbit as well, it was decided to discuss all observations afresh.

Using all observations given in the *BDS* and *ADS*, and a few more recent ones, some of which were kindly communicated by Dr. Aitken, the position angles were plotted against the time, and from their repetition a new value for the period was derived. Paying attention especially to observations during the critical time of apparent periastron, the value best fitting the observations was found to be 11.52 years—nearly the same as that of Glasenapp. All observations were then reduced to one epoch, viz., from 1900.0 to 1911.52, and combined into normal places; these are given in the first five columns of Table I. It should be added that entries 1–9 and 16–17 represent the means of a large number of nights and observers, while 10–15 represent largely single entries from the *BDS*, sometimes even single nights. The change in position angle was then so rapid that it was not feasible to take means. Furthermore, although the averages are given to $0^{\circ}.1$, not even the whole degree is certain in most cases. This is seen from an inspection of the original observations which in some years reveal systematic differences between Greenwich and Lick of as much as 20–30°. Any orbit derived from the values tabulated here must necessarily be considered as provisional. One suggestion concerning the appearance of these systematic differences may be mentioned, viz., if there is an appreciable difference in spectral class between the components (possibly a whole unit, or even more), atmospheric dispersion may play havoc with observations when the pair is very close. Thus, in apparent periastron the companion is very nearly due north of the primary, and the distance is probably not more than $0''.06$. The difference in latitude between Greenwich and Lick, coupled with the fact that in crucial years the pair may occasionally have been observed in considerable hour

angles, may well have produced systematic displacements of as much as $0''.01$ or $0''.02$. In the instance of a very close binary of widely different spectra one might even take advantage of this; such a pair might appear single in the zenith, but measurable near the horizon.

When the observations are plotted, they define a beautiful ellipse; but this is only the beginning of the trouble, for this ellipse does not obey the law of areas. As probably the position angles are more accurate than the distances, only these have been used in deriving the orbit. The graph of θ against t was drawn, $d\theta/dt$ read off, this again plotted against t and integrated with a planimeter, as a check, to see if the correct values of θ were reached at the proper times. This procedure is a little safer, since the operation with a planimeter is considerably more accurate than graphical differentiation. After fitting and refitting the values of $d\theta/dt$, and the corresponding values of r calculated from them, a passably good ellipse was obtained. The scale of this ellipse was then adjusted in such a way that the observations of distance at apparent apastron were best represented, and those at secondary maximum distance less well. The elements of the orbit, in van den Bos's notation, follow:

$P=11.52$ years	$n=31^\circ.25$	$a=0''.22$	$\omega=131^\circ$
$T=1909.86$	$i=+109$	$e=0.30$	$\Omega=111$

The representation of the observations by these elements and those of Lewis is graphically shown in Figure 1, and numerically shown in Table I, the calculation from Lewis' orbit having been made with the aid of the Thiele constants given by van den Bos.[6] Since the period is so short, this table may also serve as an ephemeris. The different columns give, in order, the number, date, and phase counted from 1900.0, of the normal place, the adopted means of θ and r, the calculated positions on these same dates from my orbit, with their O−C, and the O−C for Lewis' orbit. The last column contains the radial velocity of the companion relative to the primary, calculated from my orbit with $p=0''.0247$, as explained below.

The O−C are systematic and far from satisfactory, but both the region around apparent apastron and the crucial position angles near apparent periastron are much better represented by my orbit than

[6] *B.A.N.*, **3**, 149, 1926.

by Lewis'. It is quite possible that an orbit better than the present one could be derived from the observations, but one cannot help feel-

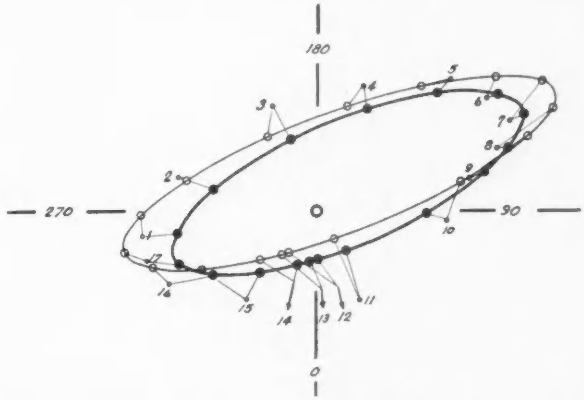

FIG. 1.—The visual orbit of κ Pegasi *AB*. Small open circles represent the normal places derived from the observations; full dots lying on the short, heavy ellipse represent the positions calculated from the present orbit; large open circles joined by the long, thin ellipse represent the positions calculated from Lewis' orbit.

TABLE I

No.	Date	Phase	Observed		Luyten		O−C		Lewis		V_{rel}
1......	1900.00	0^{P}000	278°7	0″20	278°	0″162	+ 0°7	+0″04	− 0°3	0″00	−22.9
2......	00.87	.076	256.7	.16	257	.121	− 0.3	+ .04	− 0.3	+ .01	−14.3
3......	01.93	.168	203.8	.13	201	.084	+ 2.8	+ .05	−10.2	+ .03	− 4.4
4......	02.92	.254	159.0	.15	153	.126	+ 6.0	+ .02	− 4.0	+ .03	+ 3.3
5......	03.92	.340	134.5	.21	134	.190	+ 0.5	+ .02	− 3.5	+ .03	+ 9.4
6......	05.01	.435	123.8	.23	122	.245	+ 1.3	− .01	− 2.2	− .02	+14.6
7......	06.03	.523	115.0	.24	115	.259	0.0	− .02	− 4.0	− .05	+17.6
8......	06.99	.606	109.0	.21	109	.236	0.0	− .02	− 5.0	− .08	+17.9
9......	07.65	.664	101.7	.18	102	.197	− 0.3	− .02	− 8.3	− .07	+16.1
10......	08.36	.726	85.2	.15	88	.127	− 2.8	+ .02	−18.8	− .02	+ 9.4
11......	09.06	.786	25.0	.11	38	.056	−13.5	+ .05	−32.0	+ .07	− 2.1
12......	09.28	.805	16.6	.....	3	.054	+13.6	......	+42.0	......	− 6.8
13......	09.32	.809	4.8	.....	357	.056	+ 7.8	......	+44.0	......	− 7.8
14......	09.42	.818	342.0	.....	342	.064	0.0	......	+32.0	......	−10.1
15......	09.72	.844	321.0	.13	318	.093	+ 3.0	+ .04	+25.0	− .01	−16.6
16......	10.18	.884	295.8	.18	302	.137	− 6.2	+ .05	+ 6.8	− .01	−24.0
17......	1910.78	0.936	286.4	0.20	292	0.169	− 5.6	+0.03	+ 4.4	−0.02	−27.1

ing that the visual micrometric observations have come to the end of their tether; in the last section of this paper some suggestions are made for a possible solution.

From his spectroscopic observations Henroteau gives the following values for the velocity of the system of the spectroscopic binary *Aa* at three epochs, and for the fainter visual component at one date, fortunately near the node; the velocity of this latter star appears to be constant.

TABLE II

Date	V_B	γ_{Aa}	Calc.	O−C
1900.7	....	− 2.8	− 2.6	−0.2
1912.7	....	3.8	4.2	+ .4
1917.6	+3.1	−14.6	−14.5	−0.1

Combining the orbital elements derived above with the observed relative radial velocity of +17.7 km/sec. in 1917, we find the sign of the inclination positive,[7] and a parallax of 0".0247, hence $a=8.82$ astronomical units. The mean of four trigonometric values of the parallax—rejecting the early Sproul measure and using the later determination—is 0".024, when use is made of van Maanen's recent systematic corrections.[8] The agreement, if any, is too good, which is not often the case with such a difficult object as a close visual binary. Using this value of 0".0247, we may further calculate the true velocity of the center of mass of all three stars and the mass ratio $(A+a)/B$, and find:

$$\Gamma_{\text{true}}=-8.1 \text{ km/sec.} ; \qquad (A+a):B=0.635:0.365 .$$

The agreement with the observed values of γ_{Aa} is given in the last column of Table II and is entirely satisfactory.

Using Lewis' orbit and his own spectroscopic observations, Henroteau calculated the following anomalous values for the masses:

$$A+a=10.3 \qquad \text{and} \qquad B=4.0 ;$$

the present orbit yields the values

$$\text{Mass } (A+a)=3.3\odot ; \qquad \text{Mass } B=1.9\odot ,$$

which, considering that the semi-axis major of the orbit is uncertain by at least 5 per cent and hence the masses by at least 15 per cent,

[7] This is not a new result, of course, but was known from Henroteau's observations. It was not, however, given by him, though it is stated by Beer to be positive.

[8] *Ap. J.*, **78**, 189, 1933.

appear entirely reasonable, since the absolute magnitudes are 1.8 and 2.5, respectively. For the convenience of future spectroscopic observers the relative radial velocity (B relative to A), calculated on the basis of the present data, has been added in the last column of Table I.

The results derived by Henroteau in the short-period spectroscopic system have already been commented upon; since it is highly improbable that changes in $a \sin i$, e, and ω could be observed in as short an interval as 17 years, it was assumed that all spectroscopic observations would fit one orbit. The period derived by Henroteau is accurate enough to reduce all observations to one cycle, for which the mean epoch of all observations was taken around J.D. 2419052. Corrections for the heliocentric and astrocentric light-time have been applied; to all observed epochs all observed velocities have been corrected for the velocity of the entire triple system, -8.1 km/sec., and for the orbital velocity of Aa around this center of gravity. For this latter correction the calculated values in Table II were taken; these, therefore, differ slightly from those used by Henroteau. The calendar dates of the observations, the helio-astrocentric phases and velocities thus obtained, are given in the first three columns of Table III; the geocentric J.D. and the observed, uncorrected velocities are given in Henroteau's paper and may therefore be omitted here. The second decimal place in the velocities, retained throughout by Henroteau, has been dropped here, as it is meaningless.

When these data are plotted, they lie upon a smooth curve; even the solitary observations of 1896 and 1905 and the four of 1904 fit well, as can be seen from Figure 2. This might be interpreted as evidence in favor of the present visual orbit, were it not for the fact that the visual phases of these observations do not differ greatly from those of the three well-observed epochs. The only radically discordant observation is that of 1897, which cannot be reconciled with any set of elements. If, however, the date were in error by 1 or 2 days—or, since the period is practically 6 days, by 7, 13, 19, etc., days—it would fit very well; either July 18 or July 19 for this observation would reduce the $O-C$ from more than 30 km/sec. to less than 3.

The velocity-curve is nearly sinusoidal, but the minimum is slightly sharper than the maximum, while the ascending branch is less steep

than the descending, hence e is small and ω is in the second quadrant, rather closer to 180° than to 90°. Measures with a planimeter yield the following elements:

$$\Delta\Gamma = 0.0 \text{ km/sec.}; \; K = 41.7 \text{ km/sec.}; \; e = 0.03; \; \omega = 145^\circ .$$

Since e is so small, the time of periastron passage is practically meaningless, and, instead, the instant of nodal passage may be given, the ascending node being better defined by the observations than the descending one. The representation of the observations is shown in

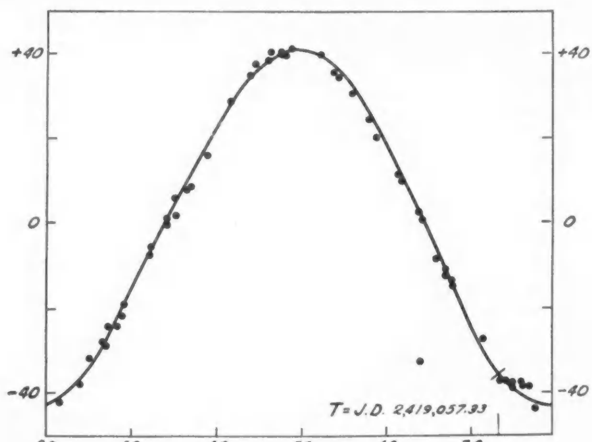

FIG. 2.—The radial velocity-curve of κ Pegasi A. Abscissae give helio-astrocentric phases counted in days from J.D. 2419050. Ordinates give the observed velocities corrected for orbital motion in the visual orbit and for the systemic velocity of AaB.

Figure 2—where T has been entered, as calculated from ω—while the actual differences $O-C$ are given in the last column of Table III. The root-mean-square deviation is 1.66 km/sec., while the average for Henroteau's three orbits is 0.99. Taking into consideration, however, that Henroteau used nearly three times as many independent variables, the mean error for one plate becomes 1.75 for the present solution and 1.25 for Henroteau. Even the first value is not excessive, considering the disturbing influence of the spectrum of the faint visual component; moreover, it is significant that of the forty-five deviations given by Henroteau, only four change sign in the present solution.

It cannot be denied, however, that the deviations from my curve are systematic in character, and still better agreement might well be reached by improving the elements with a least-squares solution. Considering the uncertainty of the visual orbit and the consequent uncertainty in the corrections to the observed velocities, such a pro-

TABLE III

Date	Phase 2419050+	V_{cor}	O−C	Date	Phase 2419050+	V_{cor}	O−C
1896 Aug. 31	2.725	−24.2	+2.6	1912 Sep. 20	7.704	−38.5	+3.8
1897 Jul. 17	6.400	−32.6	.....	23	4.753	+40.5	+0.4
1900 Aug. 6	4.600	+38.3	−0.1	29	4.774	+39.9	−0.2
7	5.595	+30.5	−1.4	Oct. 9	2.818	−24.0	−0.5
8	6.606	− 8.5	−0.9	10	3.668	+ 8.0	−3.5
12	4.638	+40.3	+1.4	11	4.785	+39.2	−0.9
21	7.623	−38.2	+3.1	13	6.783	−15.2	+0.3
22	2.657	−27.9	+1.3	Nov. 14	2.887	−21.8	−0.9
Sep. 24	5.789	+24.4	−1.5	1917 Jul. 27	5.361	+35.2	−2.1
25	6.702	−11.2	+0.7	29	7.358	−37.1	−0.9
Oct. 16	3.902	+15.8	−4.4	30	2.378	−38.1	−0.8
23	4.882	+41.1	+0.2	Aug. 1	4.388	+34.4	−0.1
24	5.894	+20.2	−2.1	3	6.149	+11.8	−0.6
1904 Jul. 6	7.495	−38.4	+0.4	5	2.155	−42.1	−0.9
Aug. 23	7.757	−43.2	−1.0	6	3.202	− 7.5	+0.5
Sep. 27	6.769	−13.6	+1.5	6	3.413	+ 0.6	0.0
Oct. 17	2.900	−19.0	+1.4	8	5.208	+39.5	0.0
1905 Jul. 24	7.420	−37.4	0.0	9	6.388	+ 2.4	+0.3
1912 Jul. 6	3.518	+ 1.5	−3.0	10	7.143	−27.0	+2.8
7	4.454	+37.9	+0.8	12	3.414	− 0.7	−1.3
18	3.506	+ 5.4	+0.8	13	4.157	+29.3	+0.6
Aug. 3	7.604	−37.5	+3.4	14	5.428	+34.6	−1.4
4	2.505	−31.4	+2.6	15	6.424	+ 0.6	0.0
17	3.702	+ 8.7	−3.7	27	6.189	+10.2	−0.7
20	6.700	−12.2	−0.3	28	7.499	−37.9	+1.5
Sep. 15	2.721	−27.9	−0.7	30	3.234	− 5.5	+1.2

cedure does not now seem warranted, especially since, e being small, the usual formulae fail, as T and ω are not now independent. K and P may well be taken as accurately known, and a solution made for δe and $\delta\omega$ in the following way: with e small we may write $\delta T = (1-2e)\delta\omega/n$ and $\delta v = (2e-1)\delta\omega$, hence

$$\delta V = K \cos \omega \cdot \delta e - Ke[\sin \omega + 2 \sin (v+\omega)]\delta\omega \,.$$

Using ten normal points, we obtain

$$\delta e = +0.0006 \pm 0.007 \,, \qquad \delta\omega = +3{^\circ\!\!.}2 \pm 8^\circ \,, \qquad \text{giving } \delta T = +0{^d}05 \,.$$

These mean errors being derived, of course, from all individual (O—C), the actual mean errors may well be much larger. By taking the horizontal deviations from the curve a correction of $+0^{d}.00002 \pm 0^{d}.00002$ to the period is derived; finally, from the O – C in Table III a mean error of ± 0.4 km/sec. is estimated for K. The elements finally adopted are given in the summary at the end of the paper; it has not been considered worth while to recalculate the O – C as the improvement on Table III must be very slight indeed.

With $P = 5^{d}.97$ and mass $(A + a) = 3.3$, we find for the semi-axis major of the relative spectroscopic orbit 14.3×10^6 km. The motion of A alone gives $a_1 \sin i = 3.43 \pm 0.03 \times 10^6$ km. Stebbins'[9] photo-electric observations indicate that no eclipse takes place (variation less than 0.02 mag.), hence on any reasonable assumption concerning the sizes of the components, i should be less than 82° and thus the minimum value of a_1 is 3.46×10^6 km., from which the maximum mass ratio is found to be 3 : 1. It seems more reasonable in the present case to assume that the two components of Aa fit on the mass-luminosity curve rather than that the two orbits are coplanar; in this case masses of 2.1 and 1.2 with absolute magnitudes of 2.0 and 4 appear reasonable.

Where the radii of the spectroscopic components are probably only 0.1 and 0.05 of their mutual distance, it is entirely possible that their individual rotations have broken away from tidal control; but unless these axial rotations are much faster than their revolution, the resultant rotation of the line of apsides must have a period of some five thousand years or more, and where e is only 0.03 it may well remain unobservable.

Combining all published observations of the optical companion C, we find no reason to doubt rectilinear motion, and there is no evidence for any perturbation, as has sometimes been assumed. Taking C to be at rest, the proper motion of AB is calculated at $\mu_\alpha \cos \delta = +0''.0303$ and $\mu_\delta = +0''.0149$, according well with that given by Boss.

After five complete revolutions of the visual system, micrometric observations of κ Pegasi are still incapable of giving a reliable orbit. This does not mean that all the elements are uncertain, for the period may well be considered as known while a comparison of the various orbits indicates that Ω cannot differ by more than 1° or 2° from the

[9] *Pub. Washburn Obs.*, **15**, 97, 1928.

value now adopted; a may be uncertain by 10 per cent, while e, i, and ω and hence T are very uncertain. In order to obtain a reliable determination of these remaining elements, the following observations are suggested:

1. Interferometer observations during one complete period, but especially during the times of small separation, in 1936.5 and 1943.9, preferably in low northern latitudes, such that the star can be observed near the zenith.

2. Intensive spectroscopic observations of A, at least once more, in order to determine e and ω accurately. Beyond this, the present elements indicate that five or six well-distributed plates at each opposition and during the full visual period would suffice to determine $a \sin i$, e, and ω in the visual orbit with great accuracy. The existing visual observations would then be called upon only to confirm the present value of Ω and to determine i, and would thus remove the last shred of uncertainty concerning the masses of this interesting system, so long considered anomalous.

SUMMARY

The various data derived for the orbits and the individual components may now be summarized.

κ Pegasi. 21:40+25:11 (1900) 4.8 F2, 5.3

Visual Orbit AB	Spectroscopic Orbit A, $A+a$
$P=11^{y}.53$	$P=5^{d}.97152\pm0.00002$
$T=1909.86$	$T=$J.D. 2419054.957 (⊙, *)
$a=0''.21$	$K_1=41.7\pm0.4$ km/sec.
$e=0.30$	$e=0.031\pm0.007$
$\omega=131^\circ$	$\omega=148^\circ\pm8^\circ$
$i=+109$	$a_1 \sin i=3.43\pm0.03\times10^6$ km
$\Omega=111$	$f=0.045\odot$

$p=0''.0247$; $\Gamma=-8.1$ km/sec. ; $\text{Mass}_B:\text{Mass}_{A+a}=0.36:0.64$

Mass $B=1.9$	$M_B=2.5$
Mass $A=2.1$ (est.)	$M_A=2.0$
Mass $a=1.2$ (est.)	$M_a=4$?

UNIVERSITY OF MINNESOTA
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THE ORIGIN OF THE GALACTIC ROTATION AND OF THE CONNECTION BETWEEN PHYSICAL PROPERTIES OF THE STARS AND THEIR MOTIONS[1]

By GUSTAF STRÖMBERG

ABSTRACT

The origin of the rotation of the galaxy and of the spiral nebulae in general is traced back to a time when the nebulae were recently formed from a common system of primordial gas. At this stage the diameters of the nebulae were of the same order as their mutual separations, the system as a whole had gravitational instability, and the mutual attractions and relative motions of the extended nebulae produced large angular momenta, but only very small angular motions. During the process of contraction in any nebula, the linear and angular velocities increase, leaving the total angular momentum constant.

The connection between mass and velocity dispersion is accounted for by the formation of massive bodies in the denser regions and less massive bodies in the less dense regions of the galaxy. The velocity dispersion also depends on the place in the system where the objects were formed.

The physical basis for the decrease in group motion with increasing velocity dispersion is found in the accumulated effect of viscous forces of different, specified types in the gaseous nebula, which includes a great number of small condensations. In the dense central part of the galaxy, viscous forces produce in the end rotation as a solid; farther out in the galactic plane they produce another type of stable, circular motion, in which centrifugal forces exactly balance the gravitational forces. The great preponderance of circular, "planetary" motions may be explained in this way. Stars formed in regions where the viscosity was insufficient to produce a stable state of motion, or before this state had been reached, move in ellipses with higher eccentricity, and have for this reason greater velocity dispersion and smaller group motion than stars formed in the galactic plane. The quadratic relationship found to exist between group motion and velocity dispersion is to be expected, at least as a first approximation.

The formation of planetary systems like that of the sun may possibly have been similar to that of the galaxy. If formed in this way, planetary systems should be rather common in the galaxy.

1. Determinations of the proper motions, distances, and radial velocities of a great number of stars in the galaxy have enabled astronomers to obtain a fairly comprehensive picture of the motions of cosmic objects of widely different physical properties, situated at distances from the sun up to about a thousand parsecs. The following observational facts about stellar motions have thus been found:

a) *Solar motion.*—The sun's velocity relative to the majority of apparently bright stars varies somewhat with spectral type, but is in general of the order of 20 km/sec., although for certain classes of objects in our neighborhood it is much greater. Relative to the

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 492.

globular clusters[2] or to extra-galactic objects,[3] it is 300 or 400 km/sec. and nearly in the galactic plane.

b) *Group motion.*—There exist moving groups of stars with small velocity dispersion. These groups either appear as clearly defined clusters or are spread out among a field of stars of varying motions.

c) *Preferential motions.*—The velocity distribution for any particular class of stars is seldom spherical. Certain velocity distributions (e.g., for apparently bright B stars) are simply flattened in the plane of the galaxy. Most stellar types have a velocity distribution elongated along an axis in the galactic plane, that is, they show a preference for motions along this axis. Since the velocity dispersion in general is small in comparison to the sun's motion relative to the galaxy as a whole, the majority of the stars share the high velocity of the sun relative to the galaxy.

d) *Connection with physical properties; asymmetry in stellar motions.*—When stars are grouped according to physical properties—like mass, intrinsic brightness, spectral type, length of period of light-variation—different values are found for the velocity dispersion and the group motion. As a general rule, whatever the physical properties of the stars involved, the velocity dispersion increases steadily with decreasing group motion relative to the galaxy as a whole;[4] further, the group motion is always along a fixed line in the galactic plane, nearly perpendicular to the axis of preferential motion. This phenomenon can also be described as an asymmetry in the motions of the high-velocity stars. Because of the importance of the connection with physical properties of the stars involved, and because we cannot specify any definite limit of velocity at which the asymmetry sets in,[5] the formulation in terms of velocity dispersion and group motion is sometimes preferable, even though the velocity distribution for the particular class of objects be asymmetric.

e) *Differential rotation.*—The radial velocities of relatively distant stars show a dependence on galactic longitude which can be interpreted as a consequence of approximately circular motions around a

[2] *Mt. W. Contr.*, No. 292; *Ap. J.*, **61**, 353, 1925.

[3] Hubble, *Proc. Nat. Acad.*, **15**, 168, 1929; Oort, *B.A.N.*, **5**, 239, 1930.

[4] *Mt. W. Contr.*, No. 293; *Ap. J.*, **61**, 363, 1925.

[5] *Mt. W. Contr.*, No. 332; *Ap. J.*, **65**, 259, 1927.

center in Sagittarius, the speed decreasing with increasing distance from the center in accordance with Kepler's third law. Combining this interpretation with the asymmetry in stellar motion, we find the most frequent velocity to be somewhat less than the circular motion. The differential rotation, first discovered by Oort, has been determined from distant O and B stars and from calcium clouds by Plaskett and Pearce,[6] with very consistent results.

2. When the general asymmetry in stellar motions was first found, Lindblad[7] immediately realized that, to a certain extent at least, it could be explained as an effect of galactic rotation. Stars with small velocity dispersion were assumed to have nearly circular motions, stars with high velocity dispersion represented objects moving in ellipses with high eccentricity, the dispersion arising from differences in the size of orbits and in the orientation of the lines of apsides and of the orbital plane. In later developments by Lindblad the energy and the area constants (i.e., the semi-major axes and the parameters) at a particular distance from the center were assumed to have certain distributions, the most frequent values corresponding to circular motions. Certain relations between group motion and velocity dispersion along different axes were also derived.

The complete absence of high velocities in certain directions is, of course, due to the fact that no stars permanently belonging to the galaxy can have a velocity greater than that of escape—an interpretation early recognized by Oort.

3. Since the following picture of the development of the galaxy is based on the supposition that galactic rotation actually exists, justification for this assumption is needed. There are a number of reasons for believing that the galaxy is in a state of rapid rotation. The most serious objection, on the other hand, has been raised by Eddington[8] and others: If the motion were strictly circular and followed the law $\omega^2 r^3 = \text{Const.}$, we would have $d\omega/\omega = -\frac{3}{2}\, dr/r$. Hence a group

[6] *M.N.*, **90**, 243, 1930.

[7] For reference see *Die Milchstrasse: Handbuch der Astrophysik*, **5**, 2, Kap. 7, 1933. In *Arkiv för Mathematik, Astronomie och Physik*, **23A**, No. 18, 1933, Lindblad has recently considered the effect of the mixing of stars of different velocity dispersion.

[8] *The Rotation of the Galaxy* (Halley Lecture; Oxford University Press, 1930; reprinted by Smithsonian Institution, *Report for 1931*, p. 239, 1932).

of stars with a diameter of, say, one-twelfth its distance from the center would, after eight revolutions, be spread out into a complete circle; and collections like the local system and the Cygnus cloud would be quickly dissipated and could have only a transitory existence.

A local condensation of stars will, however, exert an appreciable effect on the general motion about the large central mass, which cannot therefore be strictly circular. The problem of stability is then similar to that of the moon's orbit around the earth, which has been studied by Hill and others on the basis of Jacobi's integral. A preliminary investigation has shown that if the mass of the local system is one hundredth part of the central mass, for instance, the local system may have a maximum radius of about 0.14 of the distance to the center without disintegration taking place at its boundaries. From these considerations it also follows that the local system must have a rotation of its own to retain its integrity.[9] Hence we cannot expect to determine the general rotation of the galaxy from stars sharing the internal motion of the local system, and Redman,[10] for example, in discussing the radial velocities of late-type stars near the sun, found no indication of the general rotation. On the other hand, most of the stars and objects used in determinations of the galactic rotation cannot be regarded as belonging to the local system or else have only a small weight in the determinations. Moreover, corrections for the attraction of the local condensation of stars are necessary before a consistent determination of the galactic rotation can be expected from stars at moderate distances.

4. Although there is now little doubt that the galaxy is in a state of rapid rotation, with the majority of stars moving in nearly circular orbits, there remain several questions to be answered. First, with Eddington,[11] "we wonder how such a state of rapid revolution of the stars all in one direction round the center of the galaxy could ever have arisen." Second, we wonder, for instance, why such stars as cluster variables have high velocity dispersion and small group motion, while ordinary Cepheids have a very small velocity disper-

[9] A rotation of the local system has been found by Mineur which may possibly be reconciled with the conditions for stability (*M.N.*, **90**, 789, 1930).

[10] *Ibid.*, **92**, 107, 1931. [11] *Ibid.*, **88**, 331, 1928.

sion and all move with high and nearly equal velocities around the center of the galaxy. In general, the connection between the physical properties of stars and their motions remains practically unexplained. It is the purpose of this article to suggest an explanation which may be of help in solving this mystery.

5. Let us first look into the question of the origin of the rotation. The existence of high rotational velocities in spiral nebulae has been explained by Jeans as resulting from cross-currents in the primordial gas from which the nebulae are supposed to have been formed. As the nebula contracts, the angular momentum remains constant, except for a small part carried away in the escape of particles at the surface; but the angular and the linear velocities at a certain distance from the center must increase. Since we have reason to believe that the general development in the universe is such that a stage of high potential energy precedes a stage of high kinetic energy, we are inclined to think that in a system, the origin of which has been entirely independent of that of other similar systems, the circulations would, initially and hence forever, cancel out. If we suppose that the primordial gas from which a spiral nebula has evolved had no predominant circulation, the nebula would gradually evolve into a globular mass of gas with a density decreasing from the center outward, which would later give rise to a globular system of stars. There can be little doubt that the globular extra-galactic nebulae and the globular star clusters within the galaxy have been formed in this way, the spherical shape being determined by conditions existing previous to the formation of the stars.

To explain the existence of flattened, rotating systems like the spiral nebulae and the galaxy itself, it is not necessary to assume that the initial mass of gas had a relatively large angular momentum. A spiral nebula is not alone in the universe, and its origin is probably not independent of that of other similar systems. If it has evolved from a still greater system, its dimensions must originally have been of the same order as the mutual separations of the nebulae.[12] The system of nebulae would be gravitationally unstable, the nebulae

[12] Here and in the following we neglect a possible progressive increase in the linear scale of the universe. Such a change would greatly reduce the time scale of the processes here referred to.

would start moving, and encounters between two moving systems for which the ratio of the effective radius to the separation was not negligible would be the rule rather than an exception. Tidal forces would then give the passing nebulae large angular momenta, although the angular velocities involved would be exceedingly small. As we shall see later, the encounter which gave the galaxy its rotation must have taken place long before the majority of the stars were formed. Applying the same reasoning to the early stages in the formation of stars within the galaxy, we find that the existence of systems with large mean area-velocity (double stars, rotating stars) is actually to be expected.

6. Before proceeding further let us study the observed relations between physical properties and motion. We know that absolute magnitude is correlated with average peculiar motion, thus indicating a correlation between mass and velocity dispersion. This tendency toward "equipartition of energy" has been extensively studied by Seares,[13] who finds it quite marked for most of the stars for which we have sufficient data. Seares calls attention, however, to the fact that the equipartition of energy is not applicable to all classes of stars—a contention supported by later observations. For example, faint O and B stars, which are certainly very massive objects, have, according to Plaskett's work, a rather large velocity dispersion. Further, the cluster-type variables, as well as the long-period variables with periods between 150 and 200 days,[14] have very large velocity dispersion, whereas the ordinary Cepheids have extremely small velocity dispersions. We have no reason to assume that the former have extraordinarily small masses, while the Cepheids proper, according to all evidence, have smaller masses than the O stars. But in practically all cases, at least all those involving field stars, the quadratic relationship between group motion and velocity dispersion is valid within the observational errors. Consequently, we must look for an explanation of a relationship between the motions of galactic objects and physical properties besides mass.

7. The correlation between mass and velocity dispersion is often thought of as due to the accumulated effect of stellar encounters.

[13] *Mt. W. Contr.*, No. 226; *Ap. J.*, **55**, 165, 1922.

[14] Merrill, *Mt. W. Contr.*, No. 264; *Ap. J.*, **58**, 215, 1923.

But the extreme rarity of close encounters between a particular star and any other object of similar mass in the galaxy makes the time required for establishment of equipartition of kinetic energy (the time of relaxation) extremely long, much longer than the assumed lifetime of most stars. But if we go back to the time before the stars were formed, it is not difficult to understand the connection between mass and velocity dispersion. As we shall see later, viscous forces will produce equality of mean motion in very extended regions. Hence, the more massive a star, the less, in general, is the chance that its motion should differ from that of similar stars formed from matter in adjacent regions. The explanation is based on the conception that parallel, or nearly parallel, motions cannot be acquired after the stars have been formed, but must be connected with their previous history of development. We can even think of an approach to equipartition of energy arising from encounters between stars while still in an extremely diffuse state, with radii not negligible in comparison with their mutual separations.

8. But let us now study the galaxy as a whole, neglecting all condensations except the central one due to the general falling-off in density with distance from the center of the system. If at this stage stars were already formed, they would have radial motions through the center, although they would in general be deflected when passing near the center, because of small irregularities in density.

What happens now when a system of similar mass passes by? Let us first see what happens to stars already formed, which can move through the tenuous gas practically without obstruction, except perhaps near the dense central region.

Figures 1–3 show for three typical cases the computed effect of the perturbing forces acting on a star due to a passing body of mass equal to the central mass of the disturbed system. The eccentricity in the hyperbolic orbit is taken equal to 2, but the actual value is of little importance. Figure 1 shows the case for which the star has a maximum distance from its massive primary equal to one-half the smallest separation of the two great masses; in Figure 2 the greatest distance is twice this separation. In both cases the elliptic orbit of the star, after the disturbing body has receded, has a very high eccentricity (greater than 0.8). The case shown in Figure 3 represents an

original maximum distance for the star equal to one and one-half the minimum separation of the massive bodies, but the phase of the star in its orbit differs from that shown in Figure 2. In this case the star is captured by the passing mass and its final orbit around the latter is an ellipse with an eccentricity of 0.94 and a semi-major axis equal to 3.26 times the minimum separation.

We thus see that if the stars were formed before the encounter the resultant orbits would be highly eccentric ellipses and the motion in

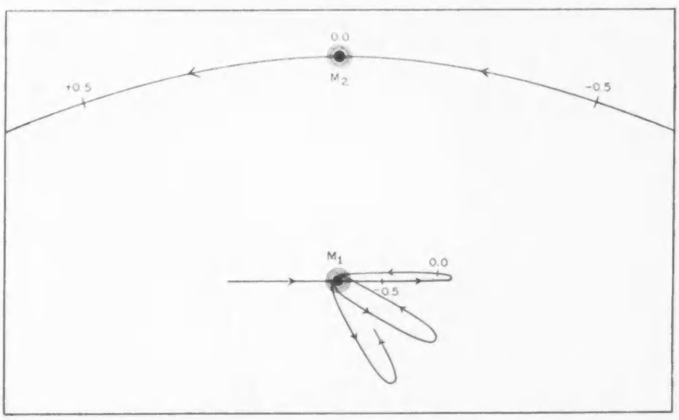

FIG. 1.—A body of negligible mass performs originally straight-line oscillations through the nebula M_1. Another nebula M_2 of the same mass passes by in a hyperbolic orbit, the numbers representing the times. The smallest distance between the two nebulae is equal to twice the amplitude of the original oscillations. The gravitational forces of the passing nebula change the straight-line motion into motion in an ellipse of high eccentricity, the elliptic motion being direct, that is, in the same sense as the motion of M_2 relative to M_1.

these ellipses would be in the same direction as the relative angular motion of the two massive bodies. Since an object in the galaxy moving in such a highly eccentric orbit would, except for a short interval when it is close to the central mass, have a very small motion around the center of the galaxy, we can say definitely that, if the rotation of the galaxy was produced by an encounter with a similar system, the only individual dense objects which could possibly have existed at the time of the encounter would be objects later developing into cluster-type variables, long-period variables with periods between 150 and 200 days, and other stars of very high velocity. The

reason for this conclusion is that when a star has once been formed as a fairly dense mass, it is subsequently little affected by collisions, by friction, or by pressure gradients in the highly tenuous system of gas.

9. If dense condensations were not already formed at the time of the encounter, the conditions are different. This preliminary survey gives only a general line of reasoning, which, in the present case, affords a more general view of the problem than can be obtained from rigorous mathematical formulations based on a number of assumptions and crude approximations. We may regard the system as a

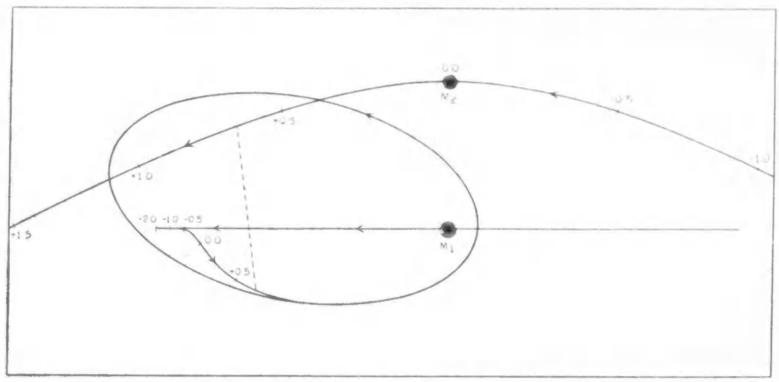

FIG. 2.—Similar to Fig. 1, but with a minimum distance between the nebulae equal to one-half the amplitude of the original straight-line motion. The dotted line indicates the smallest distance between the disturbed body and M_2. The final orbit is an ellipse with high eccentricity, and the motion is again direct.

very extended, tenuous gas originally having a spherical form with a density increasing inward. We shall assume that the gas consists of particles, each of radius r and density ρ, and that the density of the system in the region studied is $\bar{\rho}$. The mean free path is then $r\rho/3\sqrt{2}\,\bar{\rho}$. If we put $\rho/\bar{\rho}=10^{26}$ and $r=10^{-8}$ cm, we find a mean free path of 0.1 parsec. In this case collisions are fairly frequent, and the gas is extended by gas pressure. If the particles have radii of the order 1 mm, the mean free path is of the same order as the dimensions of the system and the gas is to a great extent kept extended by free-particle motion. It is clear that, whatever the nature of the particles, their average velocities must be higher near the center than in the outer regions, and hence that the temperature and the pressure must increase inward.

Since the particles are built up of electric charges, a certain part of their kinetic energy is lost in the collisions in the form of radiation. This radiation is of low frequency on account of the small energy of motion and will be spread out into space without affecting the motions of other particles. The loss of kinetic energy results in a diminution of the total energy and a general contraction of the system. Since the mean free path near the center is shorter and the mean velocity greater than in the outer regions, the time between collisions is shortest near the center, and the reduction in kinetic

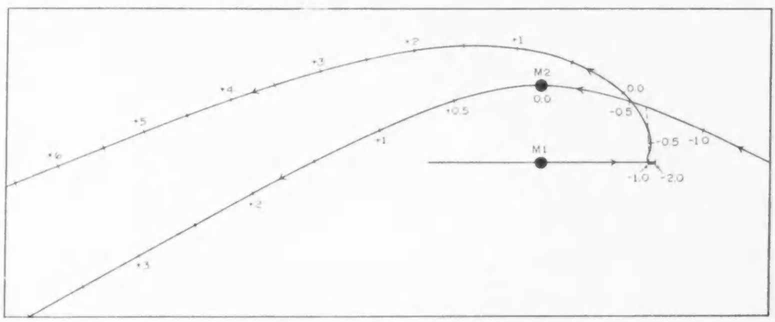

FIG. 3.—Similar to Fig. 1, but with a minimum distance equal to two-thirds the amplitude in the straight-line motion. The body of negligible mass is captured by the passing nebula. The final orbit is a large ellipse of high eccentricity around M_2, the motion here also being direct.

energy per unit mass occurs at a faster rate in the central region than at the boundary. Hence, the central part contracts faster than the outer part, thus producing an increase in the density gradient. Another cause contributing to an increase in the central density is the tendency of atoms in the outer, cool parts to combine and form heavier particles which sink into denser regions, and to lose energy in the form of radiation during the process of combination.

The tidal forces due to the passing nebula, obviously very small near the center and increasing outward, produce a greater change in velocity in the outer regions than near the center. The slow rotation caused by the passing nebula is therefore characterized by systematic transverse velocities which, although increasing as we go outward, are everywhere very small. If the rotation were one of constant angular motion, every element of the gas would have a rota-

tion, relative to an inertial frame, equal to that of the system. In any small element of the gas the local centrifugal forces, combined with the pressure gradient, would exactly balance the local attraction. If the local angular velocity were numerically greater than the general, there would be a tendency toward dispersion of the matter; if it were numerically less than the general, the tendency would be toward concentration. Since a concentration of matter would lead to the attraction of more particles to the region, the conditions would be unstable and result in a non-uniform density superimposed on the general density gradient. We would also expect that light atoms, and atoms having no tendency toward combinations, would be preponderant in the outer parts of the system.

The increase in central density would not affect the motions of the outmost parts of the system, but, for a gaseous element farther in, the central attraction obviously would be increased. It would also cause an increase in the rotational speed at a given distance from the center, since there would then be more mass inside a given sphere. The system would no longer rotate as a solid, since the angular velocity would now increase inward. The increasing rotation in the interior would also produce a progressive flattening of the whole system. It is important to note, however, that the light gas originally in the outer parts of the spherical nebula would still be found in the outer parts of the flattened nebula. The outward forces which balance the central attraction would still consist largely of gas pressure, but parallel to the plane of rotation centrifugal forces would gradually replace gas pressure. As new condensations were formed, they would tend to sink down into the equatorial plane instead of into the central region, thus contributing greatly to the general flattening of the system.

In the meantime, viscous forces of different types would be at work. In one sense a "gas" consisting of stars as molecules has an extremely high viscosity. It is, as Eddington[8] says, "the stickiest stuff there is." The reason is that the mean free path is extremely long, and momenta can be exchanged freely through surfaces following the general motion. But we are not now interested in the physical properties of "star gas," since in the end we shall be studying the motions of individual stars and star groups, not integrated effects of the motions of all stars in a given volume of the galaxy. The

effects of internal friction in "star gas," so far as it affects the history of a particular star or of an identifiable group of stars, is negligible.

Nevertheless, viscosity must play an extremely important rôle in the development of the galaxy. The molecular kinematic viscosity must be large, since a particular atom or combination of atoms may move for many years before it hits another atom or molecule. For adjacent volumes of the gas, if sufficiently large, the effect of the long free path is obviously to produce equality of mean motion. Any tendency to form condensations with masses much less than that of stars would result in a system of condensations or regions of high density with a large-scale viscosity of its own similar to a "turbulent viscosity." Consequently, the system of atoms, combinations of atoms, and condensations with masses much smaller than that of stars form together a fluid with very high viscosity in the hydrodynamic sense.

Viscous forces always produce a dissipation of energy and, if acting alone, they would in the end cause the nebula to rotate as a solid, the final rate of dissipation of energy being zero. But owing to the gravitational attractions, the dissipation function, which in general decreases, must first pass through a definite minimum, the existence of which follows from Lord Rayleigh's[15] generalization of Korteweg's[16] theorem. The corresponding state is stable, or at least of very long duration, and the general conditions in the system during the approach to this state might well be sufficiently quiescent for the formation of large condensations, later giving rise to stars and star groups. The final state thus attained is characterized by a balance between the centrifugal forces and the central attraction and can be described as a spheroidal system of gas enveloping a thin sheet of condensations in the equatorial plane of the spheroid. For sufficiently large volumes within the system, each element has a circular motion about the common axis of rotation, which is the minor axis of the spheroid, in accordance with Kepler's third law, if the central mass is predominating ("planetary" motion).

Stars formed from the matter in the equatorial sheet after the stable state has been reached must have planetary motions in the galactic plane. Since the density in this plane is relatively high, such stars presumably would be very massive, and, for a particular region

[15] *Phil. Mag.* (5th ser.), **36**, 354, 1893. [16] *Ibid.*, **16**, 112, 1883.

in the galaxy, would also have a very small velocity dispersion, since they all have nearly equal and parallel motions.

Stars formed from matter on either side of the central sheet, as soon as they had begun to condense, would acquire a harmonic motion perpendicular to the central plane. If the condensation started after the final state had been reached, they would have the same circular motion about the axis of rotation as galactic stars at the same distance from the axis, but, in addition, a velocity component parallel to the axis of rotation. It is natural to assume, however, that stars were not formed from a gas of uniform density, but only in regions where the density is non-uniform, that is, from smaller condensations. A star formed outside the central plane must then have been "born" before the steady state was reached. In this case it could have an initial velocity differing greatly from the planetary velocity, and the velocity distribution must be asymmetric since an addition to the circular velocity must be less frequent than an equal subtraction. Hence, if stars were formed from smaller condensations, we should expect those formed from matter outside the central plane to have a greater velocity dispersion and a smaller group motion than stars formed in the plane itself. Further, the density at the place of their birth being smaller than in the galaxy, and very large regions of uniform motion not yet having been formed, such stars would also have smaller masses than galactic stars.

Coming now to the cluster-type and long-period variables with their very high velocity dispersion and small group motion, it is obvious that they must have been formed far away from the center of the galaxy. In fact, Baade[17] has found numerous cluster-type variables around the galactic pole, far outside the galaxy as we picture it today. Hence these types of stars must have been formed in regions of extremely low density, where perhaps the gas consisted of atoms of small atomic weight, or of atoms with no tendency to form heavy molecules. Many of these stars may also have been formed in intergalactic regions, having been captured by the galaxy and given a small group motion in the direction of the general rotation by means of frictional forces operating when the density of the captured body was still very small and the interstellar system of gas denser than

[17] *Mitteilungen der Hamburger Sternwarte*, **5**, No. 16, 1922; **6**, No. 27, 1926; **7**, No. 36, 1931.

now. As mentioned before, another alternative is that stars of these types already existed as fairly dense bodies at the time of the transit of the nebulae that gave the galaxy its rotation.

In the central part of the nebulae, where the density is a maximum and the density gradient approaches zero, the final rotation must obviously be nearly that of a solid body. Such a rotation has been found in the central part of NGC 4594 and in the Andromeda nebula by Slipher and by Pease.[18]

The further development of the central mass is known from the works of Roche, Poincaré, Darwin, and many others. The theory has been brought to a successful conclusion by the work of Jeans; the results are well known and can also be applied to the further development of individual stars. An interesting consequence of the present picture of development is that every star and star group must be in rotation, and every star at an early stage of development must be surrounded by a huge atmosphere in which, under proper conditions, smaller condensations may be formed. Although a certain number of stars may be formed from matter ejected by the central mass, the majority of the stars at great distances from this center have never, in the present picture, been part of the central mass.

10. The calcium clouds, which still remain in gaseous form, cannot be part of the local system, since they show so clearly the general galactic rotation. In these clouds the viscous forces have had time enough to produce nearly circular planetary motions with little velocity dispersion—an observational fact showing that turbulent motions can approach the steady state here described. From the theory we should also expect the calcium vapors to form clouds and not be uniformly distributed in the galactic plane. These clouds and the dark clouds in the galaxy seem to be the last remnants of the gaseous material from which the stars have been formed.

11. Although no attempt has been made in the present general survey to find the analytical form for the general relationship connecting velocity dispersion along different axes, group motions, and physical properties, we may infer that the present distribution of velocities is the product of two distribution functions or velocity restrictions,[4] one restricting the velocity relative to the original slowly rotating nebula, the other relative to the circular motion. The limit-

[18] *Proc. Nat. Acad.*, **2**, 517, 1916; **4**, 21, 1918.

ing values of this product for maximum and minimum velocity dispersion represent the minimum and maximum effects of viscous forces. The inference that the present velocity distribution can be thus represented leads to the quadratic relationship between group motion and velocity dispersion, which holds within the observational errors except possibly for stars belonging to the local system.

12. The application of the theory here outlined for the development of the galaxy to the development of individual stars within the galaxy leads to rather interesting results. External condensations formed after the circular motions had been established by viscous forces would at a later stage be identified with planets moving in circles about the sun. Owing to the great diversity in the orbital elements of the asteroids and the periodic comets at a given distance from the sun, these objects may correspond to stars of small mass and high velocity dispersion formed outside the plane of the ecliptic before the stable state had been reached. The particles of dust which give rise to the zodiacal light would correspond to the calcium clouds. The ring system of Saturn may be regarded as a picture of the final steady state attained when the process has proceeded undisturbed and the gaseous envelope has disappeared. Since the pole of the ecliptic is only 30° from the galactic plane, the mass condensation which passed the solar system and produced its rotation must have been moving nearly perpendicular to the plane of the galaxy. Further, since the sun's axis of rotation is inclined to the invariable plane, the gas which later developed into the solar system must have had a rotation before the encounter took place—a circumstance which greatly complicates the conditions in the solar system.

At a very early stage in the development of the stars, encounters of the type here considered were probably rather frequent. As a consequence, planetary systems similar to that of the sun, if formed in the way here described, would occur with high frequency in the galaxy.

A mathematical study of these and related problems will be published in another *Contribution*.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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THE DISTANCE OF THE CYGNUS CLOUD[1]

By W. BAADE

ABSTRACT

Values of the distance of the Cygnus Cloud have been determined from three groups of stars—eclipsing systems, long-period variables, and early B stars, each of which indicates the existence of a real cloud. The resulting distance moduli are 12.51 ± 0.15, 13.0 ± 0.2, and 12.7 mags., respectively. The Cloud stars show, however, a color excess of $+0.25$ mag.; the corresponding distance with allowance for absorption according to Rayleigh scattering is $\leqq 2630$ parsecs. The long-period variables of the Cygnus Cloud follow closely the period-luminosity relation indicated by the investigations of Merrill and Strömberg, Gerasimovič, and Gyllenberg. The four Cepheid variables within the field studied seem to be affected by heavy obscuration and apparently are more distant than the Cygnus Cloud.

During the last ten years extensive surveys of variable stars in selected Milky Way regions have been under way at a number of observatories. One of the outstanding results thus far obtained is the fact that eclipsing systems and long-period variables are the most frequent types of variable stars in low galactic latitudes. Since it is our ultimate aim to use the different types of variable stars as distance indicators, eclipsing systems and long-period variables probably will play a considerable rôle in future researches on the dimensions and structure of the Milky Way system. In the following discussion they are used in an attempt to determine the distance of the Cygnus Cloud. The resulting values are in good agreement. They are, moreover, consistent with the distance derived from the early B stars of the Cloud.

S. Gaposchkin, in a recent discussion[2] of all eclipsing systems for which complete photometric and spectroscopic data are available, called attention to the remarkable fact that in general the brighter components of eclipsing binaries belong to the main branch of the Hertzsprung-Russell diagram. The exceptional cases, which seem to be rather rare, are late-type giant systems like RZ Ophiuchi, or supergiant systems of the α Cygni type like SX Cassiopeiae. Since abnormally long periods are a typical feature of these exceptional

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 493.

[2] *Veröff. Sternwarte Berlin-Babelsberg*, **9**, 5, 1932.

systems, they can easily be eliminated from a given set of data. In what follows these systems are therefore disregarded. For the normal eclipsing systems the Hertzsprung-Russell diagram can then be used in a well-known way. Since, at least in a first approximation, the brighter components of normal eclipsing systems behave like single stars of the main branch, a fair estimate of their absolute magnitudes will be possible as soon as their spectral types are known. The method should give especially useful results when applied to a group of eclipsing systems which are at practically the same distance (star cloud).

The region selected for a test of this method is the field around 22 Cygni ($19^h52^m.3+38°13'$; 1900.0), which has been studied for variable stars by the writer.[3] To obtain a representative set of data, only the fourteen eclipsing systems within 50′ of 22 Cygni are considered here, since beyond this limit the material is incomplete. The spectra of the variables have been obtained with the two-prism Rayton-lens spectrograph at the 100-inch telescope (dispersion 572 A per millimeter at λ 4350). Although a rather narrow slit was used (0.02 mm), the exposure times remained within reasonable limits, ranging from 25 minutes for a star of magnitude 13 to 4–5 hours for one of magnitude 16. Several plates with series of standard spectra were also taken with the same equipment to serve as reference spectra for classification at the spectrocomparator.

The spectral types of the variables have been estimated independently by three observers (Adams, Joy, Humason). The agreement between the individual estimates is remarkably good, the average deviation from the mean values being of the order of a tenth of a spectral division. The relation between spectral type and absolute magnitude given in Table II of *Mount Wilson Contribution* No. 244[4] has been adopted, the means of the absolute magnitudes of the *n*- and *s*- series being used here. The data thus obtained are collected in Table I.

The absolute magnitudes deduced from the spectra are on the visual scale; moreover, they refer to the brighter components only, since in the systems here considered the secondary components are too faint to be visible in the spectra. To make them comparable with

[3] *A.N.*, **232**, 66, 1927. [4] *Ap. J.*, **56**, 247, 1922.

the observed photographic magnitudes of the variables at maximum, they must be corrected for color index and for the influence of the fainter component. The latter correction, which was computed from the data in Gaposchkin's paper,[2] is as follows:

Sp. . . .	B0	B5	A0	A5	F0	F5	G0
Δm. . . .	−0.37	−0.24	−0.18	−0.23	−0.30	−0.37	−0.44

Applying these two corrections, we obtain the absolute photographic brightness of each system as given in the fourth column of Table I.

TABLE I

ECLIPSING SYSTEMS

Variable	Sp	M_v (Br. Comp.)	M_p (System)	m_p (System)	$m-M$	P
No. 3 QX Cyg.	F1	+2.8	+2.7	14.3	11.6	0ᵈ90
5 OV Cyg.	A8	2.5	2.4	15.5	13.1	3.57
24 —.	F8	3.7	3.7	15.8	12.1	. . . .
26 NY Cyg.	A1	0.9	0.7	13.8	13.1	12.05
27 PV Cyg.	A1	0.9	0.7	13.4	12.7	1.31
28 OT Cyg.	A1	0.9	0.7	15.6	14.9	1.02
34 QT Cyg.	A5	2.0	1.9	14.7	12.8	10.0 or 3.3
39 QR Cyg.	A3	1.5	1.4	13.4	12.0	28.0:*n*
41 QS Cyg.	A0	0.5	0.3	13.2	12.9	1.04
42 OU Cyg.	F3	3.0	2.9	15.5	12.6	0.56
47 OS Cyg.	A5	2.0	1.9	14.7	12.8	4.76
55 PQ Cyg.	A8	2.5	2.4	13.8	11.4	1.21
62 NP Cyg.	A2	1.3	1.2	14.0	12.8	0.78
66 OR Cyg.	A4	+1.8	+1.7	14.4	12.7	1.10

The resulting values of the distance modulus are listed in the sixth column.

Although the material is rather limited, the frequency distribution of the distance moduli leaves no doubt that the majority of the variables are situated at the same distance, i.e., are members of a star cloud. Indeed, for eleven out of the fourteen systems the value of the distance modulus differs by less than 0.7 mag. from the most frequent value; for two other stars the differences reach 1.1 and 1.3 mag., respectively. The dispersion around the mean value is practically the same as that for stars in open clusters, the distances of which have been determined by the same method. The only outstanding case is OT Cygni, which may be a background star. If this variable is excluded, the mean value of the distance modulus for the

remaining thirteen systems is $m-M=12.51\pm0.15$; the most frequent value (as read from the frequency-curve) is 12.75. The value $m-M=12.5$ is adopted here as probably the best that can be derived from the present material.

Next to the eclipsing systems, the long-period variables seem to be the type most frequently occurring in the Milky Way regions. During recent years a number of attempts have been made to derive the absolute magnitudes of these stars from their motions in space. The results may be summarized as follows: (1) All investigators who have introduced the absolute magnitude as a function of the period[5] agree that the absolute magnitude of the long-period variables increases with increasing period (or spectral type). (2) For periods >250 days the absolute magnitudes derived by the different authors are in good agreement; for the shorter periods the scatter of the individual results is considerable, owing probably to the fact that a large percentage of the variables of this group are high-velocity stars. Since the behavior of the eclipsing variables indicates that the stars of the Cygnus Cloud form a real star cloud, the long-period variables of our survey field should show the period-luminosity relation. Moreover, the distance modulus derived from them (especially the stars with $P>250$ days) should be in substantial agreement with the value obtained from the eclipsing systems. The data concerning the long-period variables of the survey field are collected in Table II.

Gerasimovič's values for the absolute magnitudes of these stars have been adopted, since they are based on the best material available at present. To reduce his visual magnitudes to the photographic scale, the color index of Shapley and Gerasimovič[6] for long-period Me variables, $+1.4$, which is in agreement with unpublished results of the writer, has been used. The data of Table II are graphically represented in Figure 1.

If we disregard for a moment PT Cygni (marked by a cross in Fig. 1), which will be referred to later in another connection, the variables do indeed closely follow the period-luminosity relation as

[5] Merrill and Strömberg, *Mt. W. Contr.*, No. 267; *Ap. J.*, **59**, 97, 1924; B. P. Gerasimovič, *Proc. Nat. Acad.*, **14**, 963, 1928; W. Gyllenberg, *Lund Medd.* (2d ser.), No. 53, 1929.

[6] *H.B.*, No. 872, 1930.

outlined by Gerasimovič. Again, the individual values of the distance modulus leave no doubt that the variables are at practically

TABLE II

LONG-PERIOD VARIABLES

Variable	$m_{\max\ pg}$	P	M_{pg} Gerasimovič	$m-M$
No. 7 PT Cyg	16^m3	160^d	....	....
8 V 341 Cyg	14.0	351	+1.0	13.0
9 V 342 Cyg	14.4	447	+2.3	12.1
11 FZ Cyg	12.5	194	−0.7	13.2
25 NW Cyg	14.2	388	+1.5	12.7
32 V 338 Cyg	13.1	321	+0.6	12.5
51 NT Cyg	14.5	390	+1.5	13.0
52 PP Cyg	13.2	297	+0.3	12.9
67 PX Cyg	14.0	309	+0.5	13.5
68 —	16.0	407	+1.8	14.2

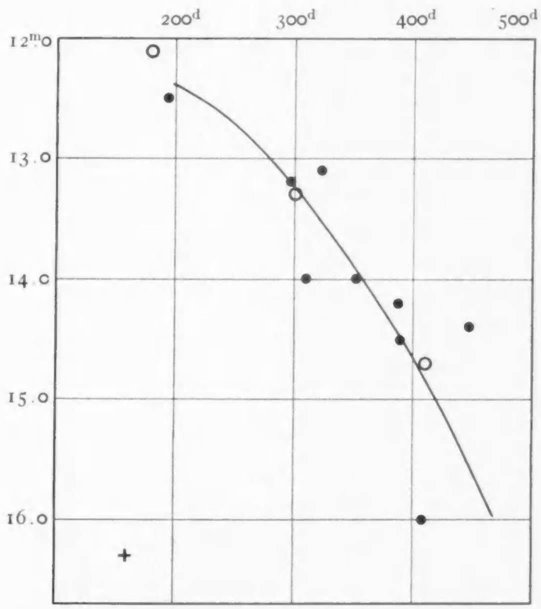

FIG. 1.—Dots and crosses, long-period variables in Cygnus field; open circles, group means from space motions (Gerasimovič).

the same distance. The adopted modulus for the long-period variables is 13.0±0.2 mag., as compared with 12.5 derived from the eclips-

ing variables. In view of the limited amount of material and the uncertainty in the zero points of both determinations, the agreement is as good as could be expected.

The early B-type stars of the Cygnus Cloud offer a third possibility of finding the distance modulus. Some years ago O. Struve[7] investigated the distribution of the O–B3 stars (mean spectral type B1) in this region. Using the data of the *Henry Draper Catalogue* and the *Henry Draper Extension*, he determined the frequency distribution of the apparent magnitudes of these stars. The outstanding feature of this distribution is a well-defined symmetrical maximum which sets in at about pg.$m = 8.0$, reaches its maximum value at 9.7, and falls off steadily for the fainter magnitudes. If the group of B-type stars represented by this maximum belongs to the same star cloud as the eclipsing and long-period variables, the distance modulus for them should be close to $m - M = 12.7$. This is indeed the case. According to recent determinations of the absolute magnitudes of early B-type stars,[8] which are in very satisfactory agreement, the mean visual absolute magnitude of B1 stars is $M_v = -3.0$. Since the stars in these investigations were selected according to apparent magnitude, this mean value needs a positive statistical correction when applied to stars distributed over a certain volume of space as in the present case. The correction—of the order of $+0.3$ mag.—largely cancels out the color-index correction when the absolute magnitude is given on the photographic scale. Therefore, with pg. $M = -3.0$ and pg. $m = 9.7$, we have $m - M = 12.7$, in excellent agreement with the values already obtained from eclipsing systems and long-period variables.[9]

Before we compute the distance of the Cygnus Cloud from the final modulus, we must take into account the fact that in this region the effects of selective absorption cannot be neglected. From the color indices of eighty stars of spectral type B8–A5 between the ninth and the thirteenth magnitudes, it was found that for the stars

[7] *A.N.*, **231**, 17, 1927.

[8] G. Strömberg, *Mt. W. Contr.*, No. 442; *Ap. J.*, **75**, 115, 1932; Öpik and Olmstead, *H.C.*, No. 380, 1933; Plaskett and Pearce, *Pub. A.S.P.*, **45**, 208, 1933.

[9] It is curious that Struve, favoring another interpretation, rejected this value of the distance modulus as *ganz unmöglich*.

of the Cloud the color excess amounts to +0.25 mag., a value closely confirmed by the color excesses of six of the eclipsing systems. Assuming pure Rayleigh scattering as the cause of the differential absorption, we can derive an upper limit for the real distance of the Cygnus Cloud. The result is

$$D \leqq 2630 \text{ parsecs,}$$

corresponding to $m-M=12.7$ as the most probable value of the modulus from our three independent determinations.

TABLE III

CEPHEID VARIABLES

Variable	m_{med}	P	$m-M$	Δ_{mod}
No. 16 V 336 Cyg	$15^{m}.82$	$15^{d}.8$	$18^{m}.3$	$+5^{m}.6$
19 GL Cyg	14.58	3.4	15.7	2.9
44 QY Cyg	15.41	3.9	16.6	3.8
56 V 343 Cyg	14.80	11.9	16.8	+4.0

In view of the consistent results derived from eclipsing systems, long-period variables, and early B-type stars, the Cepheid variables of the Cygnus Cloud need a final word of comment. At first sight the Cepheids of the survey field seem to be in hopeless contradiction to the distance modulus accepted here. If $m-M=12.7$, we should expect the Cepheid variables of the Cygnus Cloud between apparent magnitudes 9 and 12. No such variable has been discovered in the eight square-degree field around 22 Cygni, probably because the number of these stars is rather limited.[10] On the other hand, four faint Cepheids have been found in the survey field; these stars are listed in Table III. Of these four variables, V 336 Cygni can be discarded at once because it is situated in a small patch of heavy obscuration. The mean value of the distance modulus for the remaining three Cepheids is 3.6 mag. larger than that of the Cloud. Since a correction of this order to the zero point of the period-luminosity relation for Cepheids seems to be quite impossible, the only alternative left is that the Cepheids in question are not mem-

[10] A search for these variables on plates covering the whole Cygnus Cloud is under way at this Observatory.

bers of the Cygnus Cloud but are situated at greater distances. Spectroscopic observations of two of these Cepheids (GL and V 343 Cygni) by Mr. Joy strongly support this view. Both spectra show evidence of heavy differential absorption quite incompatible with the comparatively small absorption effects found for the stars of the Cloud. We therefore conclude that the faint Cepheids of the survey field are situated beyond the Cygnus Cloud, at what distance we do not know at present. Certainly a large part of the difference between their distance modulus and that of the Cloud is due to heavy obscuration. The rejected long-period variable PT Cygni may belong to this same group of stars. Inasmuch as the distance modulus for these four stars is somewhat the same, it is tempting to think of them as the brightest variables in another, more distant assemblage.

Finally, five cluster-type variables have been found in the survey field. Since the number of these stars per square degree in this field is identical with that found for high-latitude regions we feel justified in considering them as representatives of the general distribution of cluster-type variables.

A detailed account of these investigations will be published elsewhere. The writer is under much obligation to Director W. S. Adams and to Messrs. A. H. Joy and M. L. Humason for independent classifications of the spectra, and especially to Mr. Humason for help in obtaining the spectrograms of the eclipsing variables.

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ZrO IN CLASS M STARS

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ABSTRACT

Twenty ZrO bands have been found in the spectrum of β Pegasi (M2). All the stronger members of the system γ have been identified. Seven bands could not be identified with any other element or compound. The rest are blended with atomic lines, but the presence of the bands can hardly be doubted.

The behavior of the strongest ZrO band at λ 6473.7 has been studied in the brighter stars of class M. It is found to be especially strong in β Pegasi and ρ Persei. It is shown that the behavior of the ZrO band cannot be explained by the variation of either temperature or pressure. The most likely explanation is the unusual abundance of ZrO in the atmospheres of these two stars.

The ZrO bands are considered to be characteristic of the S-type spectra. In general appearance the typical S-type spectrum corresponds to early class M. In some spectra of class S the TiO bands may be present with the ZrO bands, for instance, in χ Cygni. Some spectra, for instance, HD 22649,[1] appear to be intermediate between class S and class M in the possession of bands of the two compounds.

According to H. N. Russell,[2] the relative abundance of oxygen, titanium, and zirconium in the atmosphere of the sun is of the order of 100,000:100:1. The same is probably true of most stars. In M-type stars the Ti lines of low excitation potential are among the strongest in the spectrum, while analogous Zr lines are hardly noticeable. Therefore the relative intensities of the TiO and ZrO bands in red stars depend primarily on the variation in the relative abundance of Ti and Zr rather than on that of oxygen.

The ZrO bands have never been observed in M-type stars, but this does not mean that they are altogether absent. The analogous molecules ZrO and TiO have been recently studied in detail by Miss F. Lowater.[3] The strongest bands of both molecules can be classified into three systems: α, ${}^3\Sigma-{}^3\Pi$; β, ${}^3\Sigma-{}^3\Pi$; and γ, ${}^3\Pi-{}^3\Pi$. The whole spectrum of ZrO is shifted toward the shorter wave-lengths as compared with TiO. Each of these systems consists of three heads,

[1] R. H. Curtiss, *Handbuch der Astrophysik*, **5**, Part I, 93. χ Cygni is often classified as an M-type star.

[2] *Ap. J.*, **70**, 11, 1929.

[3] *Proc. Phys. Soc.*, **41**, 557, 1929, and **44**, 51, 1932.

designated by *a*, *b*, and *c*, and each head consists of two branches, *R* and *Q*. In the case of *ZrO* it has not been determined with certainty whether the other branch is *Q* or *P*, so that, following Miss Lowater, we shall use the notation *R* and *X*. Both *ZrO* and *TiO* bands are degraded toward the longer wave-lengths. The spectra of both molecules are exceedingly rich. Miss Lowater gives the wave-lengths of some two hundred and sixty bands of *ZrO* between λ 3400 and λ 7500. Many of the fainter bands have not been assigned as yet to any system.

Since *Zr* in stars is far less abundant than *Ti*, only the bands of the lowest excitation potential will be strong enough to show on low-dispersion spectrograms. This means that if there is *ZrO* in the M-type stars, it will reveal its presence in system γ, situated in the red part of the spectrum. Systems α and β will be weak. The difference between the analogous systems of *TiO* will not be so great. This circumstance may explain the fact that the *ZrO* bands have not been found in the M-type stars. The red portion of stellar spectra, generally speaking, has not been studied in detail. Richardson[4] recently identified *ZrO* bands (mostly γ-system) in the spectra of sun-spots. We should expect, therefore, some evidence of *ZrO* in the M-type spectra.

Spectrograms of β Pegasi obtained with the self-collimating grating spectrograph in conjunction with the Perkins 69-inch reflector show many bands in the visual part of the spectrum. The strongest of these are the *TiO* systems α, β, and γ, but many fainter bands have never been reported. In the process of identification of these bands I came to the conclusion that *ZrO* is certainly present in β Pegasi. This star is usually classified as a giant of class M3. According to the Mount Wilson scale,[5] it is M2, while Russell, Dugan, and Stewart[6] give its spectral class as M5. In the strength of the *TiO* bands it is intermediate between α Orionis and α_1 Herculis, but somewhat nearer to the latter.

Two excellent spectrograms of β Pegasi obtained on October 3 and 6, 1933, have been studied in detail. They were taken on emulsion 4 F, exposures 1^h45^m and 1^h30^m. The definition is very good between λ 5200 and λ 6600. The bands in most cases show a sharper

[4] *Ap. J.*, **73**, 216, 1931. [5] *Ibid.*, **64**, 225, 1926. [6] *Astronomy*, p. 749.

edge on the side of the shorter wave-lengths. The wire of the micrometer was set on this sharp edge. The average dispersion of the spectrograph in this region is 26.7 A per millimeter. Other plates of β Pegasi give identical results.

The details of measurement are given below. In the designation of the bands their vibrational transitions (v', v'') are given first. After that follow the branch (*R* or *X*) and the member of the triplet (*a*, *b*, *c*). Then follow the wave-length and the intensity in the laboratory. When atomic lines are mentioned, their intensities in the disk (d) and in the spot (s) are given. The intensities (*i*) of stellar and of laboratory bands are of course not comparable, but they should show the same trend. When the *ZrO* bands are blended with atomic lines, a reference to the spectrograms of other M-type stars—β Andromedae (Mo), μ Cephei (M2), and ρ Persei (M4)—is often sufficient to settle the question.

γ-System of *ZrO*

(0, 0) *Ra*, λ 6229.4 (*i* 18): This band is probably present in β Pegasi. Measurement gives λ 6229.8, but it falls in the neighborhood of *TiO* band λ 6226.3. In addition to this, the *Fe* lines λ 6229.2 (d1, s2) and λ 6230.7 (d8, s10) are probably present. If there were no *ZrO* band, the two *Fe* lines should be neatly separated, which is not the case. The intensity of the *ZrO* band must be of the order 2.

(0, 0) *Xa*, λ 6235.1 (*i* 6): The stellar wave-length is 6236.1 (*i* 1); the band is very diffuse. There is not a single line in the solar spectrum between λ 6233.5 (d, -3) and λ 6236.6 (d, -3). The band is between two strong lines, λ 6233.2 *V*(d, -2; s4) and λ 6239.4 *Sc*(d, -1; s2). The discrepancy in the wave-length is not important, in view of the extreme diffuseness of the band.

(1, 1) *Ra*, λ 6260.9 (*i* 16): Star λ 6261.3 (*i* 3). Although the stellar wave-length almost coincides with *Ti* λ 6261.1 (d1, s5), there is no doubt but that the band is present in the star. The *Ti* line is very sharp, with a nebulosity on the red side. Between this line and *Fe* λ 6265.1 (d5, s8) there are no lines in the spot spectrum. The *Fe* line is strong and sharp in the star.

(1, 1) *Xa*, λ 6266.5 (*i* 6): Star λ 6266.5 (*i* 2). This is probably the

V line λ 6266.3 (−2, s4). The band is too weak to show distinctly, but the *V* line is not very sharp.

(2, 2) *Ra*, λ 6292.8 (*i* 14): Star λ 6292.4 (*i* 4). The main contributing factor is probably the *V* line λ 6292.8 (d, −2; s4), but the intensity in the star is too great for the *V* line alone. The *V* line λ 6285.1, of a slightly lower excitation potential than λ 6292.8, is only one-half as strong in the spectrum of the star.

(2, 2) *Xa*, λ 6299.0 (*i* 4): Star λ 6299.9 (*i* 1). This is perhaps Atm.o. λ 6299.2 (*i* 3). The *ZrO* band is probably very faint.

TABLE I

λ	E.P.	Intensity				
		d	s	β Peg	μ Ceph	ρ Pers
Fe 6335	2.188	6	10	8	10	5
6344	2.422	4	7	10	5	10
6355	2.833	4	6	2	5	1

(0, 0) *Rb*, λ 6344.9 (*i* 18): Star λ 6344.5 (*i* 10) diffuse. There is a neighboring *Fe* line λ 6344.2 (4, s7), but the *ZrO* band is undoubtedly the chief contributing factor in β Pegasi. The line in the star has a diffuse appendage on the side of the longer wave-lengths, whereas the next line in the spectrum of the sun is at λ 6347.1 identified with Si^+ (2 N, s0), which is certainly absent from the spectrum of β Pegasi. This band is weaker in μ Cephei and stronger in ρ Persei. The variation in intensity of the neighboring *Fe* lines of about the same excitation potential in the spectra of the three stars also confirms the identification of λ 6344 as mainly the *ZrO* band.

(0, 0) *Xb*, λ 6351.2 (*i* 6): Star λ 6351.6 (*i* 2) very diffuse. This is undoubtedly the *ZrO* band. It is greatly strengthened in ρ Persei and almost absent in μ Cephei. There are no strong lines in the solar spectrum between λ 6344.2 and *Fe* λ 6355.0 (4, s6). The latter is present in the spectrum of β Pegasi.

(1, 1) *Rb*, λ 6378.3 (*i* 16): Star λ 6378.5 (*i* 5). This band may be blended with *Sc* λ 6378.9 (s2), but the *Sc* line alone cannot account for the great intensity of the line in β Pegasi. The intensity of this

band varies from μ Cephei to ρ Persei in the same way as that of other *ZrO* bands.

(1, 1) *Xb*, λ 6384.6 (*i* 4): Star λ 6384.5 (*i* 3). The strongest line in the sun-spot spectrum between λ 6380 and λ 6393 is the unidentified line λ 6385.7 (0, s1). The *TiO* band at λ 6382.9 is too faint in β Pegasi to contribute anything here. The variation in intensity of λ 6384.8 in μ Cephei and ρ Persei is the same as for other *ZrO* bands.

(2, 2) *Rb*, λ 6412.3 (*i* 12): Star λ 6412.8 (*i* 3). The *Sc* line λ 6413.3 (s2) is probably blended with the *ZrO* band. In the star there is strong nebulosity between the presumed *Sc* line and *Fe* λ 6411.7.

(2, 2) *Xb*, λ 6419.3 (*i* 4): Star λ 6418.8 (*i* 0) very diffuse. *ZrO* is the only possibility.

(0, 0) *Rc*, λ 6473.7 (*i* 20): Star λ 6473.8 (*i* 10). There is nothing in the spectrum of the sun between *Ca* λ 6471.7 (5, s10) and *Fe* λ 6475.6 (2, s2) except atmospheric lines, the strongest of which are of intensity -1. The weakest atmospheric lines visible on the spectrograms are of intensity 2 in the sun. The *ZrO* identification is the only possibility. This band may be blended with the *Fe* line λ 6475.6, but the latter cannot be very strong, as the band is strongly degraded toward the infra-red. In ρ Persei this band is the most prominent feature between λ 6400 and *Ha*.

(0, 0) *Xc*, λ 6480.7 (*i* 4): Star λ 6481.5 (*i* 2). This is probably the *Fe* line λ 6481.9 (3, s4). The *ZrO* band, if present at all, must be weak.

(1, 1) *Rc*, λ 6508.2 (18): Star λ 6507.9 (*i* 4). The band is undoubtedly present but is blended with Atm. *w.v.* $-$ *Ti* λ 6508.1 (-3, s2). It is a diffuse band degraded toward the infra-red.

(1, 1) *Xc*, λ 6515.2 (4): Star λ 6514.5 (*i* 1). The band is probably present but may be blended with an Atm. *w.v.* (?) λ 6514.5 (2, s1). This is a real band degraded toward the greater wave-lengths.

(2, 2) *Rc*, λ 6343.0 (10): Star λ 6543.2 (*i* 3). There is an Atm. *w.v.* line at λ 6543.9 (*i* 2), but the *ZrO* band is undoubtedly present.

(2, 2) *Xc*, λ 6549.7 (2): Star λ 6550.0 (*i* 1). The band is faint but distinct. *ZrO* is the only possible identification. The atmospheric lines in this region are of intensity -3.

In addition to these, several more bands of *ZrO* have been found in the spectrum of β Pegasi. They belong to the β-system, or else are

unclassified. I shall give here only a few bands the identification of which is certain.

λ 5610.0 (*i* 10), unclassified: Star λ 5610.1 (*i* 2). This is obviously a band degraded toward the red. *ZrO* is the only possibility. This band is much fainter in β Andromedae, and is just as strong in μ Cephei.

(0, 0) *X*, λ 5633.9 (*i* 6): Star λ 5633.9 (*i* 1). *ZrO* is the only possible identification.

(0, 0) *Rc*, λ 5718.1 (*i* 20): Star λ 5717.6 (*i* 4). This band is probably blended with *Fe* λ 5717.8 (d4, s4). It looks like a real band degraded toward the red.

(0, 0) *Xc*, λ 5724.1 (*i* 12): Star λ 5724.2 (*i* 2). This band is probably blended with the *Sc* line λ 5724.1 (d, −2, s2), but in β Pegasi it looks like a band. In β Andromedae it is a sharp line.

Many more fainter *ZrO* bands are probably present in the spectrum of β Pegasi but are blended with strong atomic lines. In the foregoing discussion twenty *ZrO* bands have been identified in the spectrum of β Pegasi. Of these, seven bands could not be identified with anything else. For the rest of the bands the identification is almost as certain, as the atomic lines blended with the bands are considerably modified both in aspect and in intensity. Every *ZrO* band of the γ-system which is likely to appear in stellar spectra has been accounted for. It is evident that *ZrO* really occurs in the spectrum of β Pegasi at least.

The strongest *ZrO* band, γ (0, 0) *Rc*, at λ 6473.7, is best suited for study in different stars. It has been found on the spectrograms of all giant and supergiant M-type stars obtained at this observatory. It is certainly absent from the spectrum of Arcturus, but the neighboring *Fe* λ 6475.6 is quite prominent. Generally speaking, the intensity of the *ZrO* band increases along the sequence of the *M*-type stars, although there are striking exceptions to this rule. It is quite weak in Mo, so that the *Fe* line has only a faint appendage on the side of the shorter wave-lengths. In later subdivisions of class M the band completely absorbs the *Fe* line.

The supposed absence of *ZrO* from the M-type spectra and its abundance in class S has been much commented upon. The generally accepted explanation has been the higher temperature of the

class S stars. Not much is known about the molecular constants for *ZrO*, but it appears that the heat of dissociation of *ZrO* is about one volt greater than that of *TiO*. Therefore, under similar conditions of pressure *TiO* will be mostly dissociated while *ZrO* will still be abundant. Two papers on this subject have been recently published. Miss Cambresier and Rosenfeld[7] discuss the effects of temperature and pressure on the relative abundance of *ZrO* and *TiO* in giant and dwarf stars. Richardson[8] considers only Saha's equation. The authors of both papers postulate a uniform constitution of stars, except of classes N and R. The results are analogous, but since the first paper is based on a more general point of view, I shall consider it alone.

Miss Cambresier and Rosenfeld derive formulae for the number of molecules, based on various assumptions. Some of these may be questioned, but the general picture appears to be correct. The interesting feature of their formulae is that there is a maximum for the number of molecules depending on temperature T. According to the diagram in the article under discussion, it falls at about 2000° for *TiO* and at about 2500° for *ZrO* for the giants.

The relative abundance of *TiO* and *ZrO* for the same temperature T will be determined primarily by the relative abundance of *Ti* and *Zr* and by the heat of dissociation of the molecules. The authors assume $\mu_{Ti}/\mu_{Zr}=10$, which gives a satisfactory representation of observed facts.

It is, however, evident that the relative position of the *TiO* and *ZrO* curves is wholly determined by the assumed ratio of abundance of *Ti* and *Zr*. The point of intersection of the curves may be shifted toward lower or higher temperatures by an indefinite amount. It seems that the assumed ratio $\mu_{Ti}/\mu_{Zr}=10$ is far too low, at least for the sun. The behavior of the bands of *TiO* and *ZrO* in M-type stars cannot be satisfactorily interpreted unless the relative abundance of *Ti* and *Zr* is determined from the intensities of the atomic lines of these elements.

Even then it is possible that some of the M-class stars have a considerably higher proportion of *Zr* than others although not enough to put them into class S. The problem resolves itself into

[7] *Monthly Notices, R.A.S.*, **93**, 710, 1933. [8] *Op. cit.*, **78**, 354, 1933.

studying the behavior of bands in individual stars and not in class M as a whole. For the brighter stars of class M we have the data shown in Table II.[9]

Under the heading *TiO* the intensity of λ 6483.6, γ (4, 2) *Qc*, is given. *ZrO* means the intensity of the band λ 6473.7. It is of course not quite correct to compare the intensities of two bands which are not analogous. However, the corresponding bands of *TiO* and *ZrO*

TABLE II

Star	T	Class	R($\odot$ = 1)	Intensity		
				TiO	ZrO	ZrO/TiO
α Ori	3200°	M2	300	2	2	1.0
α Sco	3100	M1	450	5	1	0.2
β Peg	3100	M2	40	2	10	5.0
α_1 Her	2650	M5	400	10	7	0.7
ρ Per	....	M4	....	8	10	1.2

are separated by too wide an interval to afford an easy comparison. This procedure, although admittedly not rigorous, may give an idea as to the strength of the bands in the spectra of the five stars listed in Table II.

We see that two stars, α Scorpii and β Pegasi, of the same temperature differ widely in the intensity of the *ZrO* band. We should expect just the reverse effect of what is observed, namely, a much greater strength of *ZrO* in α Scorpii on account of its lower density. If we compare α Scorpii and α_1 Herculis, of approximately the same diameter, we find that the *ZrO* band is much stronger in reference to the *TiO* band in α_1 Herculis, which is again contrary to theory.

It seems that the variation of the two parameters T and g does not explain the behavior of *ZrO* in stars. There seem to be M-type stars especially rich in *ZrO*, among which β Pegasi is the most remarkable. These results agree with P. W. Merrill's studies of the S-type stars, many of which have *TiO* bands. The relative intensities of *TiO* and *ZrO* bands in this class are variable.

[9] The data for T and R are taken from Russell, Dugan, and Stewart, p. 749. The spectral class is according to the Mount Wilson classification.

It may be noted that the atomic lines of *Zr* in the spectrum of β Pegasi are entirely absent. Not a single line of *Zr* between λ 5580 and λ 6563 could be identified with certainty. Especially convincing is the absence of λ 6143.2, $a^3F_3 - z^3F_3^\circ$, which is conveniently situated for observation and is one of the strongest lines in the *Zr* I spectrum. Other strong members of this multiplet may be blended with various atomic lines. A strong diffuse line was measured at λ 5664.7 which almost corresponds to *Zr* I λ 5664.5, $a^1D_2 - y^1D_2^\circ$. This is a weak line, both in the laboratory sources and in the sun-spot spectrum, and in view of the absence of other much stronger lines, the identification would not be correct. It appears that *Zr* is wholly oxidized in the atmosphere of β Pegasi, in strong contrast to *Ti* in the same star.

Perkins Observatory
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THE PRESENCE OF SULPHUR IN THE SUN

By CHARLOTTE E. MOORE and HAROLD D. BABCOCK

ABSTRACT

With the aid of new laboratory material on analysis, and unpublished infra-red solar wave-lengths, the spectroscopic evidence of the presence of sulphur in the sun is discussed. Meissner's identifications are revised and extended. Seventeen sulphur lines are identified in the solar spectrum without question and six with some doubt.

The observable lines of ionized sulphur have such high excitation potentials that they could not be expected to appear in the sun. The coincidence of a few solar and laboratory wave-lengths is probably due to chance.

Tables give the laboratory and solar data on which the results are based.

The spectroscopic evidence for the presence of sulphur in the sun has been discussed at various times. In the *Revised Rowland Table*[1] three lines of the quintet series $4^5S^\circ - 4^5P$ in the infra-red at λλ 9212, 9228, and 9237, respectively, were entered as possible contributors to the production of solar lines.

Recently Meissner,[2] who has been working on the analysis of the laboratory spectrum, has confirmed these identifications and attributed more solar lines to *S*.

The more complete analysis of the spectrum,[3] supplemented by the work of Frerichs,[4] and improved infra-red solar wave-lengths[5] have made it possible to revise and extend the solar identifications.

The results are given in Table I, where the first five columns give the laboratory data derived from the sources mentioned above. The fourth column contains the inner-quantum numbers of the low and high levels involved in the production of the line. The rest of the table lists the solar data. In the infra-red the disk intensity is followed by the symbol ⊙ if the line is clearly identified as a solar rather than an atmospheric line. The identifications here suggested are in the last column but one. An "M" in the last column denotes that the solar identification has been discussed by Meissner.[6]

Meissner's suggestion that the group at λ 4694, $4^5S^\circ - 5^5P$, is

[1] *Carnegie Institution of Washington Publication*, No. 396; *Papers of the Mount Wilson Observatory*, **3**, 1928.

[2] *Zeitschrift für Astrophysik*, **6**, 330, 1933.

[3] Meissner, Bartelt, and L. Eckstein, *Zeitschrift für Physik*, **86**, 54, 1933.

[4] *Ibid.*, **80**, 150, 1933. [5] Babcock, unpublished material. [6] *Loc. cit.*

present in the solar spectrum is further confirmed by the behavior of the leading line, λ 4694.13, in the spot spectrum. Although a band line is to the violet, it was independently suspected that the solar line was obliterated in the spot spectrum.[7] The presence of a band line at λ 4695.452 prevents a determination of the spot behavior of the corresponding solar line, and no spot intensity can be estimated for the third and faintest member of the group.

The appearance in the solar spectrum of the infra-red triplet at λ 10455, $4^3S^\circ - 4^3P$, further supports Meissner's argument that *S* is present and shows that it is represented by more lines than was previously supposed. The second line is the weakest of the three in the solar spectrum as in the laboratory.

The presence of the group at λ 6743 is more doubtful, if the spot intensities in this region are considered. Two of the lines, λ 6743 and λ 6757, have each been assigned a spot intensity of −2N, which precludes their identification as *S*, of excitation potential 7.8 volts, since lines of this excitation would be completely obliterated. The absence of this multiplet makes the presence of the later series members extremely doubtful.

The ionization potential of an element and the lower excitation potential of a given line play an extremely important part in the identification of lines in stellar spectra. In the solar and spot spectra there is a wealth of material from which to judge the behavior of lines of elements known to be present. For example, *C*, *H*, and *O* have ionization potentials ranging from 11 to 13 volts. The Balmer lines of *H* (E.P. = 10.16) are conspicuously weakened in the spot spectrum. The two lines in the *C* multiplet near λ 4770 (E.P. = 7.45), which are not masked by lines of other elements, appear to be obliterated in the spot spectrum. The three lines of the λ 7771 group of *O* (E.P. = 9.11) are all decidedly weakened in intensity in passing from the disk to the spot spectrum.

A further illustration may be found in *Mg* and *Si* (I.P. = 7.6 and 8.1, respectively) where the ionization potentials are low enough to permit the strong spark lines to be present. *Mg* II is represented by only λ 4481 in the spot spectrum. The disk intensities of this pair are both 0, and the spot estimates −2 and "ob," respectively. In

[7] *Atomic Lines in the Sun-Spot Spectrum*, Princeton, 1933.

TABLE I

Laboratory					Sun				
λ I.A.	Int.	Low E.P.	I.Q. No.	Multiplet Desig.	Intensity: Disk	Intensity: Spot	Δλ ⊙−Lab.	Ident.	Remarks
3961.55	2	6.50	2–3	$4^5S^o-6^5P$	20	25?	−.01	*Al*	
3962.00	0	6.50	2–2	....	....	......	Abs.	....	
3962.49	0	6.50	2–1	....	....	......	Abs.	....	
4120.81	10	.....	.....	....	−1	−1	+.03	Ce^+	
4694.13	10	6.50	2–3	$4^5S^o-5^5P$	0N	ob?	−.01	*S*	M
4695.45	8	6.50	2–2	....	0N	oBd	.00	*S*	M
4696.25	6	6.50	2–1	....	−1N	......	+.02	*S*	M
4993.51	8	.....	.....	....	−3	......	+.01	*S*?	M
5278.10	0	6.83	1–0	$4^3S^o-5^3P$	....	......	Abs.	....	
5278.70	1	6.83	1–1	....	....	......	Abs.	....	
5278.99	3	6.83	1–2	....	−2	ob	−.03	*S*?	
5498.18	2	7.83	1–	$4^5P-8^5D^o$	−3	......	+.01	....	M
5501.54	3	7.83	2–	....	5	9	−.06	*Fe*	M
5507.01	4	7.84	3–	....	−3	......	−.02	....	M
5696.63	2	7.83	1–	$4^5P-7^5D^o$	−3N*	......	+.03	....	M
5700.24	4	7.83	2–	....	....	......	Abs.	....	M
5706.11	6	7.84	3–	....	0	−1N	+.01	....	M
6041.93	3	7.83	1–	$4^5P-6^5D^o$	−3	......	+.06	....	M
6046.04	5	7.83	2–	....	0N	ob	.00	....	M
6052.66	10	7.84	3–	....	−1	−1Bd	+.04	....	M
6743.58	6	7.83	1–	$4^5P-5^5D^o$	−2N	−2N?	.00	....	M
6748.79	8	7.83	2–	....	−2	......	.00	....	M
6757.16	10	7.84	3–	....	−1d?	−2N	+.04	....	M
7679.60	5	7.83	1–2	$4^5P-6^5S^o$	−3⊙	......	+.02	*S*	
7686.13	8	7.83	2–2	....	−3N⊙	......	.00	*S*	
7696.73	10	7.84	3–2	....	1	......	+.14	Atm *O* (*S*)*	
8314.73	10	.....	.....	....	−2⊙	−2	+.04	....	M
8585.60	10	.....	.....	....	−1⊙	−1	−.02	....	M
8670.19	1	7.83	1–0	$4^5P-4^5D^o$	−3	......	+.03	*S*	M
8670.65	2	7.83	1–1	....	−2N⊙	......	−.02	*S*	M
8671.37	1	7.83	1–2	....	−2⊙	......	−.06	*S*	M
8679.00	1	7.83	2–1	....	−2⊙	......	−.05	*S*	M
8679.70	2	7.83	2–2	....	1⊙	0	−.05	(*S*)*	M
8680.47	8	7.83	2–3	....	−2⊙	......	−.07	*S N*	M
8693.24	1	7.84	3–2	....	....	......	Abs.	....	M
8693.98	3	7.84	3–3	....	0⊙	0	−.02	(*S*)*	M
8694.70	10	7.84	3–4	....	1	−1	−.06	−*S*	M

* Parentheses indicate that the *S* line is masked.

TABLE I—*Continued*

Laboratory					Sun				
λ I.A.	Int.	Low E.P.	I.Q. No.	Multiplet Desig.	Intensity Disk	Intensity Spot	Δλ ⊙− Lab.	Ident.	Remarks
8874.53......	9	8.38	4−	$3^5D^o-5^5F$	−3	......	−.05	*S*?	
8879.62......	1	8.38	0−	....	......	......	Abs.	....	
8880.70......	3	8.38	1−	....	−3N⊙	......	.0	*S*?	
8882.47......	5	8.38	2−	....	−3	......	−.11	−*S*?	
8884.23......	7	8.38	3−	....	−3⊙	......	+.01	*S*?	
9212.91......	10	6.50	2−3	$4^5S^o-4^5P$	2	......	−.08	Atm *S*	M
9228.11......	10	6.50	2−2	....	−1	......	−.02	*S*	M
9237.49......	10	6.50	2−1	....	−3	......	−.1	*S*	M
10455.47......	8	6.83	1−2	$4^3S^o-4^3P$	5	......	−.02	*S*	
10456.79......	4	6.83	1−0	....	2	......	.00	*S*	
10459.46......	8	6.83	1−1	....	4	......	−.03	*S*	
11453.........	1	8.38	4−	$3^5D^o-4^5F$	......	......	†	....	
11464.........	.....	8.38	.....	....	......	......	†	....	
11472.........	.....	8.38	.....	....	......	......	†	....	

† Masked by wide groups of Atm lines (Babcock).

spite of their laboratory strength, they are faint even in the solar spectrum. *Si*, which is unquestionably present in both disk and spot spectra, illustrates clearly the behavior of lines arising from different atomic levels. The lines λ 3905 and λ 4102 (E.P. = 1.90) have the same intensities in both disk and spot spectra. The multiplet at λ 5645 (E.P. = 4.9) has all of its members definitely weakened in the spot spectrum. The strong *Si* II lines at λ 6347 and λ 6371 (E.P. = 8.08) are obliterated in the spot spectrum.

In the spectrum of the disk the principal factors which are unfavorable to the appearance of a spectral line are a high excitation potential in all cases[8] and a high ionization potential in the case of a spark line. Since the temperature of the spot is lower than that of the disk, they are still more unfavorable there. In view of the conclusive evidence afforded by these and other elements, the *S* lines of E.P. = 7.8 cannot account primarily for the solar lines λλ 6757, 6743,

[8] H. N. Russell, *Mount Wilson Contribution*, No. 383, p. 25; *Astrophysical Journal*, **70**, 35, 1929.

and 5706, all of which are present in the spot spectrum. If due to *S* alone, these lines would be entirely obliterated in this source.

TABLE II

Laboratory					Sun				
λ I.A.	Int.	Low E.P.	I.Q. No.	Multiplet Desig.	Intensity: Disk	Intensity: Spot	Δλ ⊙−Lab.	Ident.	Remarks
4142.24I... .28B..	8 8	15.78	$\frac{1}{2}$–$1\frac{1}{2}$	$4p^4D^o-4d^4F$	2	2	+.07 +.03	−*Ni?*	m*
4145.05I... .10B..	8 10	15.80	$1\frac{1}{2}$–$2\frac{1}{2}$	....	−3	....	+.04 −.01	......	??
4153.05I... .11B..	10 12	15.83	$2\frac{1}{2}$–$3\frac{1}{2}$	....	−1	1N	+.01 −.05	*Cr*	m
4162.64I... .70B..	10 15	15.88	$3\frac{1}{2}$–$4\frac{1}{2}$	....	1N	1	+.03 −.03	......	m
4168.37I... .40B..	5 8	15.80	$1\frac{1}{2}$–$1\frac{1}{2}$	....	......	....	......	......	Abs.
4189.68I... .69B..	4 12	15.83	$2\frac{1}{2}$–$2\frac{1}{2}$	....	......	....	......	......	Abs.
5428.64I... .69B..	5 9	13.53	$\frac{1}{2}$–$1\frac{1}{2}$	$4s^4P\ -4p^4D^o$	−2	....	+.08 +.03	......	Too ft.
5432.77I... .83B..	9 12	13.56	$1\frac{1}{2}$–$2\frac{1}{2}$	....	−3	−2	−.02 −.08	......	Abs.
5453.81I... .88B..	10 15	13.61	$2\frac{1}{2}$–$3\frac{1}{2}$	....	−2	....	+.05 −.02	......	??
5473.59I... .63B..	5 15	13.53	$\frac{1}{2}$–$\frac{1}{2}$	....	−2N	−1	−.03 −.07	*Ti*	m
5509.58B.. .67I...	15 5	13.56	$1\frac{1}{2}$–$1\frac{1}{2}$	....	−1	−1N, b?	−.03 −.12	......	m
5564.93B.. .94I...	8 4	13.61	$2\frac{1}{2}$–$2\frac{1}{2}$	....	−3	....	+.05 +.04	......	Too ft.

I = wave-length by Ingram. *Physical Review*, **32**, 172, 1928.
B = wave-length by Bloch, *Annales de Physique*, **12**, 12, 1929.
* m = masked.

Although the evidence shown in Table I leaves no doubt that sulphur is moderately abundant in the sun, a critical examination of

the multiplets of *S* II does not substantiate the claim that this element is observable in the ionized state, as was recently suggested.[9] The range of lower excitation potentials of the strong lines in the visible region is from 13 to 17 volts, and the ionization potential of *S* I is 10 volts. Since this amount of excitation is necessary for the production of the strong *S* II lines and since the strongest *S* I lines are very faint in the solar spectrum, the coincidence of several solar and laboratory wave-lengths is probably to be attributed to chance and not to the presence of *S* II, particularly since the laboratory measures of *S* II lines by different authors are discordant by ± 0.05 A. Table II contains the laboratory data of the leading members of two strong multiplets, and the nearest solar lines, arranged similarly to Table I. The double entries in the first, second, and eighth columns give the laboratory wave-lengths, the intensities by different measurers, and the corresponding differences in wave-length, sun *minus* laboratory.

Each multiplet must be considered as a unit. If represented in the sun, the lines of the main diagonal of the multiplet in question should be present. The high excitation required to produce these lines of *S* II prevents their appearance in the spot spectrum. With these criteria, the only possible line in the first multiplet is λ 4145, and, in the second, λ 5453, and the coincidence of wave-lengths depends on choice among discordant laboratory measures.

Similarly for the other strong multiplets, most of the leading diagonal lines are masked or absent and the coincidence in wave-length of a total of some four or five lines with inaccurate laboratory measures affords no evidence of the presence of *S* II in the sun.

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[9] Bartelt and Eckstein, *Zeitschrift für Astrophysik*, **7**, 272, 1933.

A STATISTICAL STUDY OF THE SOLAR ATMOSPHERE WITH APPLICATION TO THE EVOLUTION OF PLANETS

By DINSMORE ALTER

ABSTRACT

The paper considers the following subjects: (1) gain of moment of momentum of particles in the solar atmosphere, due to radiation pressure; (2) an assumption regarding areal acceleration due to this gain; (3) application of this areal acceleration and of observed variation of solar radiation to explain the changing coronal forms; (4) application to explain the sun's equatorial acceleration; (5) the formation of planetesimals and the partial segregation of elements in meteorites; (6) the solar system in general, especially the peculiarities of the earth-moon system and the planes of the orbits of the planets.

From these studies the author believes that it is probable that the solar system has been built without the aid of tides produced by a passing star.

The primary difficulty to be faced in any hypothesis for the formation of planets from a single star is that of the necessary increase in moment of momentum. It will be shown that this increase is secured through radiation pressure. Before beginning the mathematical derivation, it may be well to look at this qualitatively. The photons moving out from the photosphere of the sun may be considered to some extent as additional small particles of the atmosphere. As such they increase the viscosity of the gas and, therefore, tend toward producing rotation with a constant angular velocity as we go out from the sun, instead of with a constant moment of momentum.

In 1926 Jeans established, from a different analysis than the one used here, that, in the nucleus of the sun, radiation tends to equalize angular velocities of different levels. In the nucleus we have the additional factor that the radiation of higher levels on lower ones is important. This equalization he counterbalanced by assuming that the mass destroyed to produce radiation is near the sun's center, where it has a low moment of momentum. Even though this assumption be correct, it does not affect the present problem.

In the accompanying diagram, let P be a molecule in the equatorial solar atmosphere, with radius vector R and with a component of velocity, $\overline{V}$, in the equatorial plane and perpendicular to the radius vector. Let B be a radiating differential area of the photosphere,

shown at the instant of emission. P is shown at the instant of reception. ρ is, therefore, the path of the radiation. $\bar{v}$ is the velocity of B at the instant of emission. v, the speed, is a constant for all radiating areas in a given solar latitude. τ, θ, and σ are of obvious interpretation from the diagram. The limits of τ are $\pm\sigma$.

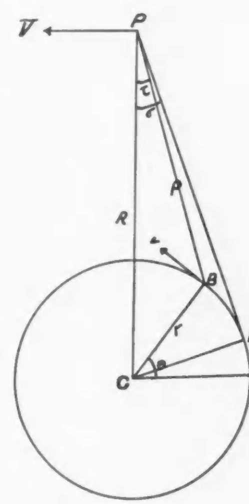


FIG. 1

From the $\triangle CPB$

$$\cos(\theta-\tau)=\frac{R}{r}\sin\tau\,.$$

For an observer on P, the source B is moving with a velocity $\bar{v}-\overline{V}$ and, therefore, has a component of velocity $[(\bar{v}-\overline{V})\cdot\bar{c}]/c$ along ρ, where the direction of $\bar{c}$ is taken as along ρ.

Therefore, he associates the frequency

$$\nu\left(1+\frac{(\bar{v}-\overline{V})\cdot\bar{c}}{c^2}\right)$$

with the radiation. The momentum is observed as

$$\frac{h\nu}{c^2}\left[1+\frac{(\bar{v}-\overline{V})\cdot\bar{c}}{c^2}\right]\bar{c}\,.$$

In scalar form this becomes

$$\frac{h\nu}{c^2}\left[c+\frac{v}{r}R\sin\tau-V\sin\tau\right]$$

and the component in the direction of $\overline{V}$ is

$$\frac{h\nu}{c^2}\left[c\sin\tau+\left(\frac{R}{r}v-V\right)\sin^2\tau\right].$$

It is difficult to integrate the effect over the whole sun for two reasons: first, an angle of equal altitude as seen at the particle above the solar equator does not intersect a constant solar latitude; second, the angular velocity of solar rotation is not a simple function of latitude. Indeed, the rate is not known at the poles.

We shall, therefore, merely integrate the effect of a strip of constant width around the equator, throwing the width into the general constant of the radiation, and writing $d\tau$ as the differential area.

The total component in the direction of $\overline{V}$ becomes, therefore,

$$\frac{kh\nu}{c^2}\int_{\tau=-\sigma}^{+\sigma}\left[c\sin\tau+\left(\frac{R}{r}v-V\right)\sin^2\tau\right]d\tau=\frac{kh\nu}{c^2}\left(\frac{R}{r}v-V\right)(\sigma-\tfrac{1}{2}\sin 2\sigma)\ .$$

This becomes zero when P is part of the radiating surface and when it is at an infinite distance. It also becomes zero when P and B have equal angular velocities and negative when P has the greater angular velocity.

The term $(\sigma-\frac{1}{2}\sin 2\sigma)$ decreases from $\pi/2$ at the radiating surface rapidly toward zero at infinity.

ASSUMPTION REGARDING ACCELERATION

The gain in moment of momentum shown by this equation is rigorously derived. However, its interpretation in terms of acceleration involves an assumption. Under the assumptions usually made in relativity discussions, P will have approximately a constant areal velocity. However, these assumptions are based on a strict analogy between photons and material particles which have a finite rest mass. There appears to be no observational evidence to invalidate a different assumption.

It appears to be entirely permissible to assume that the gain in moment of momentum will increase the areal velocity, and this assumption will be made here for the case where P is an atom or an aggregate of a few atoms. Any one of the following, or possibly others, would lead to such a result: (1) That the momentum of a photon is greater than its mass times its velocity. (2) That the velocity and dynamical directions of a photon differ by an angle not greater than the angle of aberration due to the relative transverse velocities of the emitting and receiving particles. (3) That not all of the mass of the photon is conserved when it excites or ionizes an atom, and that there is no compensating mass added to the electric field under such conditions. If none of the mass be conserved, there

will be a gain in areal velocity, so long as the angular velocity of P is less than that of the photosphere.

These possible explanations of the assumption, although not identical, are not necessarily independent of one another.

An interesting corollary of the extreme case of each of these is to reduce the Doppler and aberration effects to perfect symmetry, so that if an apparatus could be devised for measuring accurately the directions of recoil particles, the aberration measured would give the relative transverse velocities. The writer has been unable to devise any other critical experiment to test his assumption.

A second consequence, independent of the main purpose of this paper, is that the loss of mass of a star due to the mass-energy relationship becomes secondary to that due to repulsion of atoms into interstellar space for a rotating star, and that it thus reconciles stellar ages with expansion data.

A third appears to be that non-rotating stars might develop into planetary nebulae, and those of extreme rotational speed into white dwarfs. More study must be given to this last point.

The writer pleads as justification of his assumption the demand by it for certain observed facts of the solar system that so far have been difficult to explain by other means. These are developed in detail as the main body of the paper.

EFFECT DUE TO SCATTERING

The equation, as derived, gives the effect of absorbed radiation on P. However, in the outer corona, especially, much of the light is scattered. To study the effect of such non-absorbed light requires a more tedious though not difficult calculation. To save space this calculation will be omitted here. The result is $4/3$ of that due to absorption.

RATE OF GAIN OF MOMENT OF MOMENTUM

This $\overline{V}$ component of the radiation pressure becomes zero when $v=V$ and $R=r$; in other words, when the particle is part of the radiating surface. This condition is approximated as soon as the atmosphere of the sun takes on any considerable opacity, probably a few hundred kilometers below the photosphere. From there outward it increases to a maximum value, then decreases to zero asymptotically.

For absorbed radiation, the $\overline{V}$ component of the momentum of the radiation is

$$\frac{(\sigma-\frac{1}{2}\sin 2\sigma)\left(\frac{R}{r}v-V\right)}{c}$$

of the total momentum. For a particle in the upper atmosphere for which there is a balance between radiation and gravitation, or for which the radiation pressure is the greater, the gain in moment of momentum under the extreme form of assumption is quite rapid. Let us assume such a balanced particle at $R=2r$, with initial moment of momentum equaling that of a similar particle on the photosphere.

$$\sigma=\frac{\pi}{6}\ ;\qquad \left(\frac{R}{r}v-V\right)=\tfrac{3}{2}v\ ;\qquad v=\frac{c}{145{,}000}\ ;\qquad G=6800\ \text{cm/sec.}^2\ .$$

Therefore, the $\overline{V}$ component of its acceleration:

$$V'=6800\cdot\frac{3}{290{,}000}\left(\frac{\pi}{6}-\frac{1}{2}\sin\frac{\pi}{3}\right),$$

$$=0.0064\ \text{cm/sec.}^2\ .$$

We find that this particle will gain at the rate of doubling its original moment of momentum in 181 days. Considering the small effective weight of molecules in the atmosphere, the gain in R must be quite rapid.

If we consider a molecule similar to the last, but at a distance $R=4r$, we find that the acceleration in the direction $\overline{V}$ has been reduced to 0.12 of the value at $R=2r$. At $R=10r$ it is reduced to 0.022 of the value at $R=2r$.

This acceleration reaches a maximum (using units of $r=1$) at

$$\frac{R^2\sqrt{R^2-1}}{R^2+1}=\frac{\sigma-\frac{1}{2}\sin 2\sigma}{1-\cos 2\sigma}\ .$$

This is roughly at $R=1.192r$. A particle similar to the one above, but at this distance, is gaining moment of momentum at a rate to double it in 82 days instead of 181 days as at $R=2r$.

APPLICATION TO CORONAL VARIATIONS

The corona is observed at times of sun-spot minima to extend in long streamers in, or roughly parallel to, the equatorial plane. At times of sun-spot maxima it is uniform with respect to latitude; there are no long streamers and the inner corona is much brighter than at times of minima. At sun-spot maxima the intensity of solar radiation is several per cent greater than at minima.

Even when the radiation is a minimum, the observed density of the reversing layer, near the photosphere, is about 0.0001 of the earth's atmosphere. This is interpreted as indicating a very close balance between gravitation, G, and radiation pressure, L, for the mean molecule of the atmosphere. However, if at times of minimum radiation we do have such a balance, the increased thermal agitation at times of maximum radiation will carry enough additional molecules through the photosphere to tend toward preventing a lowering of the pressure near it, even though L becomes decidedly greater than G for the mean molecule.

If we have an outward acceleration, we find that any two molecules, each starting with an initial radial velocity v_0 through the photosphere and one a distance d below the other, will have this distance increased at any chosen level by an amount which depends on the acceleration. Therefore, when the radiation pressure is at a maximum the outward rate of decrease of density must also be a maximum, and no matter how much greater the density of the corona may be near the photosphere for an acceleration $\alpha_1 > \alpha_2$, there must be some higher level at which the density is less with the acceleration α_1 than it is with α_2.

This gives us exactly what we do find. With the increased radiation, an additional number of molecules are forced through the photosphere, giving the intense inner corona which is characteristic. The radial spreading, which is due to the increased acceleration, forbids the long streamers.

At times of minimum radiation there is a much closer balance between G and L, with G perhaps predominating. If so, molecules will not be carried to great heights above the photosphere. However, the $\overline{V}$ component of the radiation must increase their moments of momentum. The form of the equation shows that this effect is a maxi-

mum at the equator and declines to a minimum of zero at the poles. We do have, therefore, a gradual outward motion of the molecules, predominating in the equatorial plane. The velocities acquired will be much smaller than those due to an excess of perhaps 3 per cent of L over G, at times of maxima. Therefore, there will be a lesser flux of molecules through the photosphere, but a far slower equatorial rate of decrease outward of the coronal density. This demands the fainter inner corona but the much longer equatorial streamers which characterize the minima.

It is perhaps needless to point out that with the $\overline{V}$ term decidedly smaller than the $(G-L)$ term at times of maximum radiation, there can be but little latitude difference in the corona. That $(G-L)$ may merely tend toward zero near the epoch of minimum radiation is indicated by the fact that the coronal type, associated with maximum radiation, predominates during the majority of eclipse observations, even to within a year or so of the minimum.

THE EQUATORIAL ACCELERATION OF THE SOLAR ROTATION

Let us consider any level of the solar atmosphere. Atoms are colliding at all sorts of relative velocities. If, at a low-enough relative velocity, two atoms collide which have sufficient chemical affinity, or if two collide at a relative velocity lower than that for escape from the solid state, they will adhere. The total gravitational force is unchanged, but the sum of the radiation pressures on the two separately is greater than that on the aggregate, therefore they must fall in very sharply toward a perihelion. Because they have gained moment of momentum, their velocities at perihelion will be greater than that of the average molecule at the level. The collisions at these lower levels must, therefore, tend to increase the angular velocity. Our equation shows that the greatest gain in angular velocity is in the equatorial plane, therefore the equatorial rotation speed of the sun must be increased over that at higher latitudes. It has been rather commonly stated that this acceleration is quite probably a residue of an effect at some earlier stage of the sun's history. It requires but little reflection to be certain that such an effect would be damped out quickly by viscosity and by circulation of the gases, if the cause did not persist.

FORMATION OF PLANETESIMALS

If the atoms which adhere have gained sufficient moment of momentum, their perihelion will lie high in the solar atmosphere and they will follow a more or less permanent orbit about the sun. Jeans has developed the equation

$$M=\left(\tfrac{1}{3}\pi\kappa\right)^{\frac{3}{2}}\cdot\frac{c^3}{\gamma^{\frac{3}{2}}\rho^{\frac{1}{2}}},$$

where c is the molecular velocity, γ the gravitational constant, ρ the initial gas density, κ the ratio of specific heats, and M the minimum mass of a condensation which will be permanent. From this equation, and from fairly reasonable assumptions, he has secured the right orders of mass for galaxies and stars, a spectacular and valuable piece of work. He applies the equation to the atmosphere in the neighborhood of a star and attempts to show by it that the planets must originally have been of approximately their present masses. It seems to the writer that in this final conclusion he has overlooked a very important physical condition. Let us consider some certain atom, say iron, at a height where the black-body temperature would be, say, 500° C. Two atom aggregates will form. These, of course, may be considered as a molecule of the atmosphere. But it is also legitimate to consider them in their relationship to each other as a solid particle. The question whether the number of such aggregates formed will exceed the number destroyed by bombardment from high relative velocity atoms becomes one which may be considered under equilibrium vapor pressure of a solid and will require a far less density of the gas than for the condensation of a gas to attract other atoms into it and cause further condensation by their mutual attractions. Even applying Jeans's own equation, c^3 has become practically zero and ρ for this aggregate is large.

Assuming that the number of two-atom aggregates can increase, we shall find no difficulty in the formation of planetesimals. Before considering their formation in detail we shall examine two observations which indicate the existence of such a formation.

In meteorites we find a peculiar segregation of the elements, a segregation which appears, to a lesser extent, in the earth's crust.

Let us consider certain molecules, high in the solar atmosphere and, for simplicity, consider them as moving in the same circular orbit, which lies in the equatorial plane. Let m be the moment of momentum of one of them, C the centripetal force, and μ the mass.

$$(G-L)\equiv\frac{K}{R^2}=C=\mu\omega^2R=\frac{m^2}{\mu R^3}=\frac{\mu V^2}{R}=RGV^2\ ,$$

or

$$KR=\frac{m^2}{\mu}=(G-L)R^3\ .$$

Let

$$\zeta\equiv\left(\frac{G-L}{G}\right)\ .$$

In general ζ will be different for each chemical element.

$$V=\sqrt{\frac{\zeta}{R}}\ .$$

Since R is the same for these molecules, the velocities in the orbit must vary as $\sqrt{\zeta}$.

Therefore, those atoms which have a nearly common value of ζ will most often collide with a small relative velocity.

We find, therefore, that it is much more probable that two similar atoms will collide at a low-enough relative velocity to adhere than that two dissimilar ones will do this. Between atoms for which there is a strong chemical affinity, dissimilar atoms may unite on collision, especially where such stable compounds as water are formed. Therefore, although there is no separation in the atomic state other than that atoms with a small positive or a negative value of ζ will be relatively more abundant in the chromosphere and inner corona than in the solar nucleus and outer corona, the aggregates must tend toward a segregation of the elements, exactly as is found to be the fact. The chemical similarity and the nearly identical atomic weights of Fe, Ni, and Co become significant factors in producing the mixtures of these, so common to meteorites.

The second piece of observational evidence of formation of such

aggregates is that Encke's comet is perturbed at perihelion, in such a manner that it must have encountered a resisting medium. This is the only short-period comet with a perihelion inside of Mercury's orbit.

Molecules, such as H_2 and He, will move out rapidly from the sun. Owing to their high equilibrium vapor pressure, aggregates cannot be formed permanently until they are far outside the earth's present orbit. Therefore, except as they form compounds, they will not be gathered into the planetesimals. The chemical inactivity of He, therefore, results in very little of this common element existing on the earth, although the still lighter H_2 is found abundantly.

Larger aggregates must begin to appear as soon as two-atom aggregates can continue to exist. At first these will fall back quite sharply toward the sun, owing to the additional decrease of L. However, the percentage change in L becomes less for each additional atom, and, consequently, the necessary gain in moment of momentum before this higher aggregate can become semi-stable is decreased. Eventually the aggregates may be considered as planetesimals. Such consideration is preferable as soon as the addition of molecules to the aggregates decreases L by a negligible amount.

It is probable that the very great majority of atoms escape into space instead of forming aggregates. The hypothesis indicates, therefore, a more rapid loss of solar mass than that computed from the mass-energy equation. If this be true and the ages of stars are less than are now assumed, one of the greatest objections met by advocates of the expanding universe is eliminated.

THE SOLAR SYSTEM

Once planetesimals have been formed, their grouping into planets follows the process described by Chamberlain and Moulton. These authors have discussed this sufficiently that it need not be considered here in any detail.

The gravitational force between these aggregates in contact is small, but, in comparison with the sun's tidal force tending to separate them, it is tremendous, so that gradually the size of the planetesimals must increase. Any which attain a size considerably larger than the others will increase at a much greater rate than the smaller

ones and there is, therefore, a tendency for them to aggregate into a comparatively few of large size. Collisions between the larger ones must occasionally take place, with a corresponding increase in the rate at which the new body takes on mass. In general, therefore, that one which through chance has become larger than the others will more and more dominate the swarm.

Decrease in mass of the sun, owing to loss of atoms into space and conversion of matter into radiation, will cause the swarm to move farther and farther from the sun. This will also be aided by the gain in moment of momentum through the $\overline{V}$ term of the radiation pressure. Eventually the increase of distance will take it so far that the density of the atmosphere is insufficient to add much to the swarm. We shall speak of that region where the density of the atmosphere is sufficient to add materially to the mass as the "collecting zone." The swarm, gradually collecting into one body, becomes a planet. It would seem probable that the swarm perturbing Encke's comet is in process of becoming a very small planet, within the orbit of Mercury.

In the past, the radiation intensity of the sun, its mass, and its diameter were much greater than now. Therefore, the collecting zone was much farther from the sun than at present. Matter streamed out from the equatorial plane at a much greater rate. From the observed diameters of the giant stars the sun possibly had a photospheric diameter at one time of a hundred million miles, or even more. The radius of the collecting zone may have been several astronomical units. It is easy to prove that the *latus rectum* of a planet's orbit will increase inversely as the mass of the sun.

As soon as one planet has been formed, its tidal action will increase the amount of matter carried out in the equatorial plane of the sun. We shall expect, therefore, successive planets to be larger, until the diminished solar radiation causes a rapid decrease in size. This is, in general, what we do find in the solar system. A temporary variation in the radiation would produce such an exception as Uranus and Neptune. The orbits of Mars and the asteroids interlace, and we may perhaps consider the whole group as one whose formation has been interrupted by the perturbations of Jupiter. There is no estimate of the number of small asteroids and, though usually the

limit of mass has been guessed as very much smaller, it may be that the mass of that system is greater than that of the earth. If this be true, there is only the one minor exception to the progression of mass of the planets.

The assumption of this paper resolves the difficulty which has been encountered in the direction of rotation of the planets. Let us consider a small aggregate or planetesimal, which under the method of formation described here must have an areal velocity greater than that of the photosphere. On the side toward the sun more energy is retained as heat mass than on the side away from the sun. Therefore this side must lag with respect to the opposite side, producing a spin in the observed direction.

The formation of the earth-moon system becomes a minor modification of an old hypothesis. The separation of the moon from the earth has been questioned, on account of its orbit lying quite closely in the ecliptic, despite the $23\frac{1}{2}°$ inclination of the earth's equator. If, in our case, two planets formed in the same collecting zone and later suffered a glancing collision, one of them somewhat north of the other, the friction of the collision would reduce the relative velocities sufficiently to make the system stable. The plane of the equator would be changed, but the mutual orbit plane would be little affected. Tidal force would then act, as shown by Darwin, to drive them farther apart. It is interesting to note how closely other satellites, except for the probably captured outer ones of Jupiter and Saturn, follow the plane of the primary's equator in all cases. Such peculiar rotations as those of Uranus and Neptune are explained in a much similar manner, except that complete fusion occurred.

The encounter hypothesis does not explain the fact that the orbit plane of Mercury lies very closely in that of the sun's equator and that the orbit plane of Venus lies roughly at half the inclination of the ecliptic. Since, at a time when the sun's radiative force was much greater than it is today the formation of planets was, comparatively speaking, a rapid matter, it follows that if the sun's axis describes any precession under the attraction of the galaxy, the hypothesis must demand this orbit shift for the more slowly formed recent planets.

We have, therefore, a plausible hypothesis of the formation of the

solar system, using only such phenomena as are observed on the sun today. The accidental encounter of two stars, which, according to Jeffreys, must be a grazing collision, and exceptionally rare, is not needed. If the main features of this hypothesis be true, solar systems are the common result of one stage of stellar evolution, instead of being extremely uncommon. Perhaps in a somewhat similar manner the Cepheid stage has produced binary stars.

The writer wishes to thank several friends for their extremely patient consideration of his problem and for their many constructive criticisms which are important to any success that the work may secure.

University of Kansas
August 1932

NOTES

A STRIKING CHANGE IN THE BRIGHTNESS OF PERIODIC COMET 1925 II

A comparison of Plate X, *a* and *b*, shows at a glance the spectacular change in brightness that occurred between March 10 and 14, 1934. They are enlargements, to the scale 1 mm $= 0\overset{'}{.}39$, of two plates exposed for twenty minutes at the 24-inch reflector of the Yerkes Observatory. The middles of the exposures occur on March 10.29 and 14.28, 1934. Universal Time. The stellar images are well matched in the two plates. On the first date the comet, hardly as bright as magnitude 18, shows a tiny nucleus surrounded by a faint round coma which can be traced to a diameter of $30''$ on the original plate. Four days later the diameter is reduced to $15''$ but the whole coma is bright and quite sharply defined, so that at first sight the image of the comet appears as that of a star of magnitude 13.

The orbit of Comet 1925 II (Schwassmann-Wachmann) is exceptional in character, in that it is distinctly planetary; the period is 16 years, the inclination only $9\overset{\circ}{.}5$, and the eccentricity $e = 0.135$ makes the path more nearly circular than that of Mercury ($e = 0.206$). The heliocentric distance varies only between 5.3 (1925) and 7.3 (1933), so that the comet remains all of the time between the orbits of Jupiter and Saturn. Since its discovery in 1927 this object has been observed year after year, even through aphelion—the first instance of such an occurrence. The theoretical brightness, computed from the law of inverse square distances, does not vary by more than 1.3 mag. between an opposition at perihelion and one at aphelion, and in any one apparition the theoretical change from opposition to quadrature is not more than 0.3 mag.

From the great heliocentric distance and its small percentage of change, this comet would be expected to be more inert, more inactive than any other. Yet from the beginning of the observations of this object in 1927 large fluctuations in brightness as well as in

structure have been noticed. For instance, on February 10, 1931, it had brightened up to the 13 mag. after remaining fainter than the 16 mag. for more than three months. Again on January 20, 1933, it was as bright as the 12 mag., while a month earlier the magnitude was only 17 although the theoretical brightness should not have changed by more than a tenth of a magnitude. So far we have no indication of the rapidity with which the outbursts of activity occur. The interval of only four days between the two exposures reproduced here clearly shows that the rise must be swift. The decline, on the contrary, has always been slow, the comet remaining relatively bright for several weeks before it reverts to the faint appearance which seems to be the more common aspect.

Continuous observation of this object is difficult; it remains rather close to the ecliptic and is therefore lost in the moonlight every month. The record of its appearance is at present too incomplete to establish a periodicity in the fluctuation of its brightness. It is not conceivable that internal action in this distant comet can account for such phenomena as the sudden, hundred-fold increase in brightness that we have described. Ionic streams from certain parts of the sun seem a more promising source of explanation. In this connection it is suggestive that on March 11 a heavy magnetic storm was recorded. The comet, less than a month after opposition, may have been influenced by the same source of energy. But this would hardly account for its continued bright phase, which lasted for weeks afterward. The evidence from previous bright phases is, however, not conclusive, since the dates are not accurately defined.

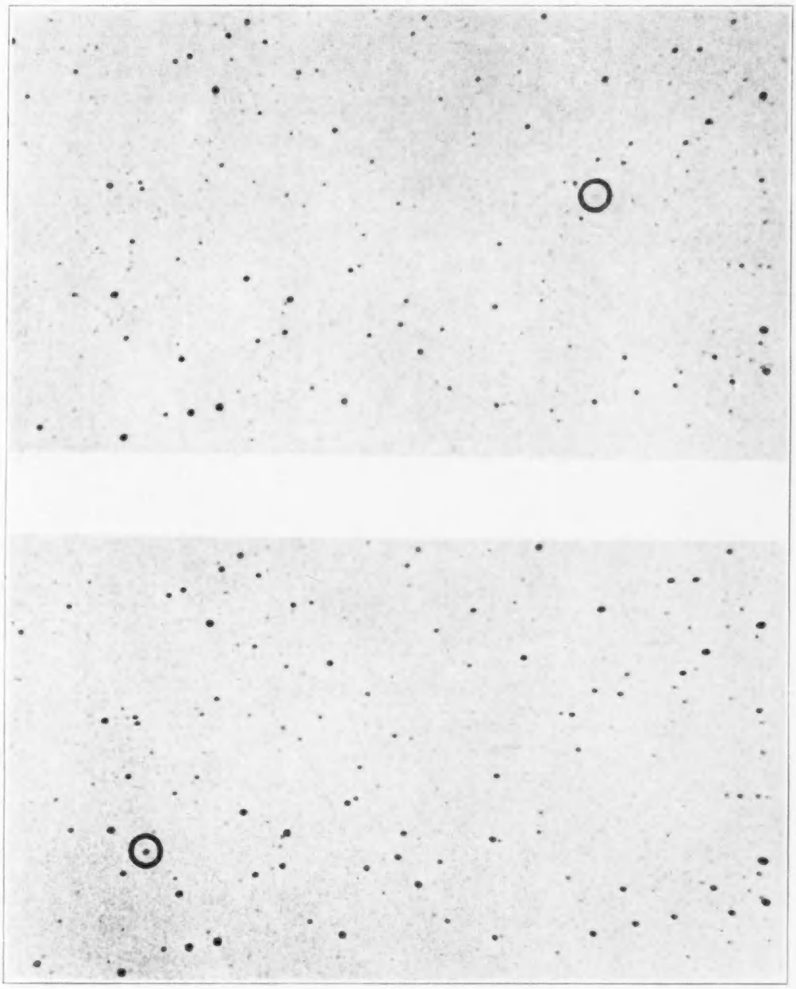
The occurrence described here typifies the difficulty in representing cometary brightnesses by any mathematical expression and the danger in drawing conclusions as to the development and possible future disintegration of comets from such data.

G. Van Biesbroeck

Yerkes Observatory
April 1934

PLATE X

a



b

Periodic Comet 1925 II

a) 1934 March 10.26956 U.T. $10^h12^m44^s + 8°29'.8$

b) 1934 March 14.27855 U.T. 10 11 14 8 36.5

TRANSMISSION COEFFICIENTS OF LIGHT-FILTERS

In the *Astrophysical Journal* for March, 1934, page 172, John S. Hall states that he used in the computation of the distribution of effective energy of stars the transmission coefficients of his filters taken from *Wratten Filters*. This booklet is a catalogue of filters supplied by the Eastman Kodak Company, and the transmission coefficients given there are intended as a guide to the choice of filters by purchasers.

The object of this note is to point out that it is not safe to rely on the transmission coefficients given in that catalogue as corresponding precisely to the transmission coefficients of a given filter purchased by number. While every care is taken to see that filters correspond to the published transmissions within commercial limits, transmission coefficients used in scientific work should be determined specifically for the filter actually employed. If this presents any difficulty to the investigator, our research laboratory will be very glad to determine the actual transmission coefficients for filters which are to be employed in scientific work.

There does not seem to be any reason to believe that the use of these transmission coefficients in any way vitiates Hall's work, but it seemed desirable to publish this warning.

C. E. K. Mees

Eastman Kodak Company
Rochester, N.Y.
March 21, 1934

A II IN THE SPECTRUM OF υ SAGITTARII

ABSTRACT

Seventeen lines in the spectrum of the peculiar supergiant υ Sagittarii agree in position with the strongest laboratory lines of singly ionized argon. Ten of these, including the two strongest laboratory lines in the observed region, are sensibly unblended.

The spectrum of υ Sagittarii (HD B8p; F2p) has been investigated in detail by J. S. Plaskett.[1] The spectrum contains outstanding lines of *Fe* II, *Ti* II, and *Cr* II together with a well-developed *He* I spectrum similar to that found in types B2–B3. The hydrogen lines are complex and the intensities of the spectral lines in general change

[1] *Publications of the Dominion Astrophysical Observatory*, 4, 1, 1926.

considerably. The nature of the intensity variations has not as yet been investigated.

TABLE I

λ (Lab.) (Rosenthal)	Term Combination	Int. (Lab.)	λ (Star)	Int. (Star)	Identification
4013.87	$3d^4D_{3\frac{1}{2}}-4p^4D_{3\frac{1}{2}}$	(10)	4013.96	1	*A* II
4042.91	$\left\{\begin{matrix}4s^2D_{1\frac{1}{2}}-4p^2D^{\circ}_{1\frac{1}{2}}\\ \text{Limit }{}^1D\end{matrix}\right\}$	(8)	4042.82	1	*A* II
4072.01*	$\left\{\begin{matrix}4s^2D_{2\frac{1}{2}}-4p^2D^{\circ}_{2\frac{1}{2}}\\ \text{Limit }{}^1D\end{matrix}\right\}$	(9)	4071.85	1:	*Fe* I 1.75?
[4103.91	$\left\{\begin{matrix}4p^4D^{\circ}_{3\frac{1}{2}}-5s^4P_{2\frac{1}{2}}\\ 4p^2D^{\circ}_{2\frac{1}{2}}-5s^2P_{1\frac{1}{2}}\end{matrix}\right\}$	(10)]	[4103.77	1:]	*A* II?
4266.53	$(4s^4P_{2\frac{1}{2}}-4p^4D^{\circ}_{2\frac{1}{2}})$	(10)	[4267.11	3]†	Masked by *C* II and S II
4331.25	$4s^4P_{1\frac{1}{2}}-4p^4D^{\circ}_{1\frac{1}{2}}$	(10)	4331.35	1	*A* II
4348.11	$4s^4P_{2\frac{1}{2}}-4p^4D^{\circ}_{3\frac{1}{2}}$	(20)	4348.04	2–3‡	*A* II (see note)
4379.74*	$4s^4P_{\frac{1}{2}}-4p^4D^{\circ}_{\frac{1}{2}}$	(8)]	[4380.13	1n]§	*A* II blended
4426.01	$4s^4P_{1\frac{1}{2}}-4p^4D^{\circ}_{2\frac{1}{2}}$	(15)	4426.02	1–2	*A* II
4430.18*	$4s^4P_{\frac{1}{2}}-4p^4D^{\circ}_{1\frac{1}{2}}$	(9)	4430.03	1	*A* II
4545.08	$(4s^2P_{1\frac{1}{2}}-4p^2P^{\circ}_{1\frac{1}{2}})$	(10)	4544.85	1‖	*Ti* II 5.15?
4589.93	$\left\{\begin{matrix}(4s^2D_{1\frac{1}{2}}-4p^2F^{\circ}_{2\frac{1}{2}})\\ \text{Limit }{}^1D\end{matrix}\right\}$	(9)	4589.99	1	*Ti* II .96
4609.60	$\left\{\begin{matrix}4s^2D_{2\frac{1}{2}}-4p^2F^{\circ}_{3\frac{1}{2}}\\ \text{Limit }{}^1D\end{matrix}\right\}$	(15)	4609.80	1¶	*A* II (see note)
4726.91	$4s^2P_{1\frac{1}{2}}-4p^2D^{\circ}_{1\frac{1}{2}}$	(10)	4726.96	1	*A* II
4735.93	$4s^4P_{2\frac{1}{2}}-4p^4P^{\circ}_{1\frac{1}{2}}$	(15)	4736.27	1**	*A* II blended? (see note)
4764.89	$4s^2P_{\frac{1}{2}}-4p^2P^{\circ}_{1\frac{1}{2}}$	(10)	4765.20	1:††	*A* II
4806.07	$4s^4P_{2\frac{1}{2}}-4p^4P^{\circ}_{2\frac{1}{2}}$	(20)	4806.03	2–3‡‡	*A* II (see note)
4847.78*	$(4s^4P_{1\frac{1}{2}}-4p^4P^{\circ}_{\frac{1}{2}})$	(8)	4847.91	1‖	*Cr* II 8.27

* Lines marked with an asterisk were suggested by Miss Moore as being likely to be present from their theoretical intensities.

† A faint line was measured at 4266.73 on one plate. It is not in general resolved from the stronger carbon line and I have preferred to give the wave-length of the latter and to consider the *A* II component as masked.

‡ At certain phases of the variation of the spectrum of υ Sagittarii this line becomes diffuse and shifts its wave-length about 0.3 A toward the red. It is possible that at these times *A* II becomes blended with a weak *Mn* II line at 4348.43. Plaskett gives the wave-length of this line from three plates as 4347.94.

§ A broad, diffuse line whose edges were measured at λ 4379.35 and λ 4380.72. A laboratory line doubtfully ascribed to *Mn* II at 4379.74 may contribute to the blend.

‖ These wave-lengths were taken from Plaskett's measures. No attempt was made to measure them on Yerkes plates.

¶ On some spectrograms this line is quite diffuse and is probably a blend.

** There is a faint line about an angstrom to the violet of the measured position. The two lines are not completely resolved and accurate bisections are difficult to make.

†† There is probably a typographical error in the line listed by Plaskett at λ 4674.83. If the second and third digits are interchanged, the wave-length agrees with the laboratory position of the argon line.

‡‡ Plaskett gives a line at 4805.39. This is probably a blend of *Ti* II 4805.11 and *A* II. On the best Yerkes plates there is a well-defined line at 4806.0 and a faint component is suspected on the violet side which is probably due to *Ti* II.

In a later note[2] Plaskett published a list of wave-lengths of the lines present in the spectrum. Many of the lines are unidentified. A

[2] *Ibid.*, p. 115, 1928.

number of the previously unidentified lines are due to *S* II, which seems to be stronger in this spectrum than in any other stellar source known at the present time. When it was noticed that the strongest *A* II line in the region of best definition coincides with an unidentified stellar line in Plaskett's list, an examination was made of Yerkes spectrograms of the star to see if other strong argon lines were present. All of the stronger lines were found to be present and the presence of singly ionized argon is demonstrated. Several of the lines are included in Plaskett's list and the others were measured on Yerkes plates. Between the limits $\lambda\lambda$ 4000–$H\beta$ there are seventeen lines in which *A* II is an appreciable contributor and ten of these lines, including the two strongest, are sensibly unblended. The data concerning the identification are given in Table I. The spectrum of *A* II has been classified by De Bruin.[3] The laboratory wave-lengths and intensities in Table I are taken from Rosenthal.[4] Struve[5] measured a faint line at λ 4348 in γ Pegasi (B2) and β Orionis (B8p) which agrees in position with the strongest *A* II line in the region of best definition on Yerkes plates. This line shows well on the reproduction of the spectrum of v Sagittarii given by Plaskett.[6]

I am greatly indebted to Professor H. N. Russell and to Miss Charlotte E. Moore for critical comments on the identifications.

W. W. Morgan

Yerkes Observatory
Williams Bay, Wis.
May 8, 1934

[3] *Zeitschrift für Physik*, **48**, 61, 1928, and **51**, 108, 1928.

[4] *Annalen der Physik*, **4**, 49, 1930.

[5] *Astrophysical Journal*, **74**, 234, 1931.

[6] *Op. cit.*, Pl. I.

REVIEWS

Astronomie: Tatsachen und Probleme. By OSWALD THOMAS. Graz and Vienna and Leipzig and Berlin: "Das Bergland-Buch" (Deutsche Vereins-Druckerei). Pp. 584; Pls. 31; Figs. 275. Unbound: RM. 3.80 (S. 7.-); bound: RM. 4.80 (S. 9.-).

This excellent popular book contains a complete account of the facts and problems of modern astronomy. Historical accounts and instrumental data have been deliberately omitted in order not to obscure the main subject of the discussion. The presentation is novel and exceedingly vivid, and many interesting illustrations add greatly to the value of the book. It is intended primarily for the amateur, but the professional astronomer will also read it with interest. The arrangement of the tables and especially the line drawings should appeal to all those engaged in the teaching of elementary astronomy. The use of mathematical formulae has been avoided, but numerical computations are scattered throughout the book. The low price of this volume is especially commendable. The paper and the printing are excellent.

O. S.

Physica. (Periodical appearing in one volume of about 960 pages yearly, in ten issues.) 8vo. Vol. I, No. 1 (December, 1933). The Hague: Martinus Nijhoff. Subscription: 25 guilders annually.

The Dutch periodical *Physica: Nederlandsch Tijdschrift voor Natuurkunde*, published since 1921, has been subdivided into two journals: *Physica* and *Nederlandsch Tijdschrift voor Natuurkunde*. The latter will be published in the Dutch language and will appeal particularly to readers in the Netherlands.

Physica is designed to present to foreign workers the contributions of Dutch physicists, many of whom have hitherto published their papers in the German journals. The articles will be printed in any one of three languages: English, French, or German.

If the first number may be regarded as a good indicator, *Physica* will be welcomed as one of the most important periodicals in the field of physics. The wide range in material is suggested by this issue, which contains

papers on the emission of light in gas discharges, the construction of a quartz fluorite achromatic lens, isotopes of lithium, a sum rule in photography, Tchebycheff polynomials, and other subjects.

P. C. K.

Physical Optics. By ROBERT W. WOOD. 3d rev. and enlarged ed. Pp. xvi+827. Figs. 462. Pls. 17. New York: Macmillan Co., 1934. $7.50.

It is hardly fair to the book to call it merely a "new edition." The second edition appeared in 1911 and much of the material in it has been omitted; in fact, only about half of the old edition has been retained and many of the chapters have been re-written. Of chief interest to the physicist, however, is the fact that the book has been enlarged from 695 to 827 pages, and that 495 pages consist of new matter.

In view of the growth of spectroscopy since 1911, a new chapter on "The Origin of Spectra" was obviously necessary, and this chapter gives the introductory theories of line and band spectra. Use is made later of this material in the chapters on "Resonance Radiation and Fluorescence of Atoms," "The Resonance and Fluorescence Spectra of Molecules," "The Fluorescence and the Phosphorescence of Liquids and Solids," "Magneto-optics," "Electro-optics," and "The Raman Effect."

The chapters on "Relativity" and "The Nature of White Light" in the second edition have been omitted. It is to be regretted that the author omitted the fundamental equations of diffraction for which the reader is referred to the second edition.

The book includes a description of the experimental technique and results of many of the author's investigations. It is characteristic of Wood that he endeavors as always to give whenever possible a physical picture of what is happening instead of relying merely on equations. Equations are freely employed, however, whenever they appear to be necessary.

This book will be read with interest and with profit by all who have an interest in physical optics.

HENRY G. GALE

INDEX TO VOLUME 79

SUBJECTS

INDEX TO VOLUME 79

AUTHORS